Due Tu 10/08/13 (at the beginning of the class)

1. Use the (matrix) exponential series to evaluate e^{At} for:

   (a) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$;

   (b) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

   Also, determine the eigenvalue decomposition (i.e., the eigenvalues and eigenvectors) of these two matrices.

   Note: You are not allowed to use Matlab in this exercise.

2. Suppose $A(t)$ is an $n \times n$ time-varying matrix with continuous entries that satisfies

$$A(t) \left(\int_{t_0}^{t} A(\sigma)\, \mathrm{d}\sigma \right) \;=\; \left(\int_{t_0}^{t} A(\sigma)\, \mathrm{d}\sigma \right) A(t).$$

   Show that the state-transition matrix $\Phi(t_1, t_0)$ can be computed as

$$\Phi(t, t_0) \;=\; \exp\left(\int_{t_0}^{t} A(\sigma)\, \mathrm{d}\sigma \right).$$

3. Find the state transition matrix $\Phi(t_1, t_0)$ for the matrix

$$A(t) \;=\; \begin{bmatrix} \alpha(t) & \beta(t) \\ -\beta(t) & \alpha(t) \end{bmatrix}$$

   where $\alpha(t)$ and $\beta(t)$ are continuous functions of t.

4. For the Mathieu equation,

$$\ddot{y}(t) \;+\; (\omega \;-\; \alpha \cos(2t))\, y(t) \;=\; 0$$

   use Matlab to compute the state-transition matrix with $x_1 = y$, $x_2 = \dot{y}$, $\omega = 2$, $\alpha = 1$. You should do your computations on the time interval of length equal to three periods of oscillations for $t_0 = 0$ and $t_0 = 1$.