Consensus and vehicular formulation

12-05-11

- use spatially-invarient theory to answer some of the questions that arise in these problems.

- These systems can be thought as spatio-temporal systems, where signals depend on time and discrete spatial variable 'n'.

- Coupling between subsystems can come either from mathematical modeling or from distributed control at the level of control objective.

- optimal Control of <u>finite platoons</u>

$$J = \int_0^\infty \left(p^T(t) Q_p p(t) + q_v v^T(t) v(t) + r u^T(t) u(t) \right) dt$$

$\to 1$ (if Lead fict. Veh. is removed)

$$Q_p = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$\to 1$ (if follow fict. veh. is removed)

when Lead & follow fictitious vehicles are added to formation.

If both Lead & follow fictitious vehicles are removed Q_p becomes singular.

- <u>infinite platoons</u>

$$\begin{bmatrix} \dot{p}_n \\ \dot{v}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_n \\ v_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n \quad ; \quad n \in \mathbb{Z}$$

$$J = \int_0^\infty \sum_{n \in \mathbb{Z}} \left((p_n(t) - p_{n-1}(t))^2 + v_n^2(t) + u_n^2(t) \right) dt$$

$\downarrow$ Spatial Z_θ - transform

$$A_\theta = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad Q_\theta = \begin{bmatrix} 2(1-\cos\theta) & 0 \\ 0 & 1 \end{bmatrix}$$

$$0 \leq \theta < 2\pi$$

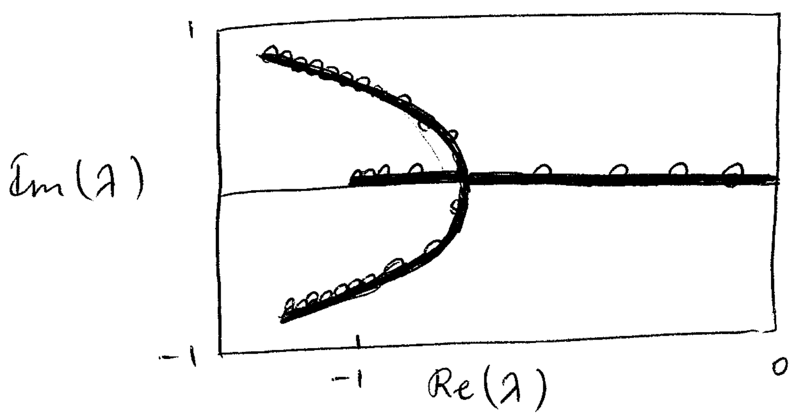
- The coupling comes from the performance index.

☒ if $\theta = 0$, $Q_p = 2(1-\cos\theta) = 0$
meaning that the pair (A_θ, Q_θ) is not detectable.

☒ fix: penalize global position errors:

$$Q_p = q + 2(1-\cos\theta)$$

$q = 0 \Rightarrow$ many modes have slow rate of convergence.



thick solid line → infinite platoons

Symbols → finite platoons

Q_p is symmetric - Toeplitz matrix

$$Q_p = V \Lambda V^* \quad ; \quad V V^* = I$$

$$q_p(\theta) = 2(1 - \cos\theta)$$