$$\begin{cases} f(x) \approx P_N(x) \\ f(x_i) = P_N(x_i) \quad ; \quad i = \{0, \dots, N\} \\ x_i \quad \dots \quad \text{interpolation points} \end{cases}$$

$$f(x) = P_N(x) + \underbrace{R_N(x)}_{\text{Remainder}}$$

$$f(x_i) = P_N(x_i) + \cancel{R_N(x_i)}^{\,0}$$

<u>So</u>

$$R_N(x_i) = 0$$

Why is this important?

$$f(x) = \sum_{n=0}^{N} a_n \phi_n(x)$$

$$P_N(x) = \sum_{n=0}^{N} b_n \phi_n(x)$$

$$a_{m,G} = \langle \phi_m, f \rangle_G = \sum_{i=0}^{N} w_i \phi_m(x_i) f(x_i)$$

at x_i we have $f(x_i) = P_N(x_i)$

$$= \sum_{i=0}^{N} w_i \phi_m(x_i) P_N(x_i)$$

G Stands for Gaussian Quadrature

$$= \langle \phi_m, P_N \rangle_G = b_n$$

<u>Therefore</u>

if we decide to compute <u>spectral coefficients</u> a_n with Gaussian Quadrature, there is no difference or error between a_n and b_n. So, we can leave $f(x)$ and work with $f(x_i)$ or $P_N(x_i)$ with no error.