09-08-11

Examples of distributed Systems

$$\dot{x} = Ax + B_1 d + B_2 u$$

x ... state

d ... disturbance

u ... input control

$$\phi_t = \phi_{xx} + d \qquad (*)$$

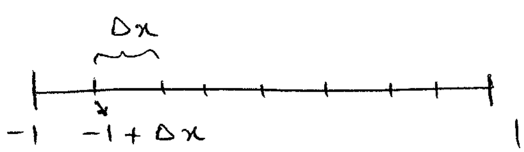
$$\phi(x=-1,t) = u(t)$$

$$\phi(x=1,t) = 0$$

Approximate second derivative with central difference

$$\left.\frac{\partial^2 \phi}{\partial x^2}\right|_{x=\bar{x}} \simeq \frac{\phi(\bar{x}+\Delta x) - 2\phi(\bar{x}) + \phi(\bar{x}-\Delta x)}{2\Delta x}$$

This yields a finite dimensional approximation of $(*)$ with the state given by



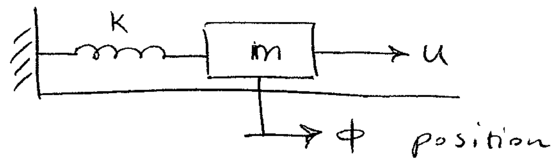
$$\phi_n = \phi(x = -1 + n\Delta x)$$

$$\Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix}$$

$$\dot{\Phi} = A\Phi + B_2 u + B_1 d$$

$$A = \frac{1}{2\Delta x}\begin{bmatrix} -2 & 1 & 0 & 0 & \\ 1 & -2 & 1 & 0 & \\ 0 & 1 & -2 & 1 & \\ & & \ddots & \ddots & 1 \\ & & & 1 & -2 \end{bmatrix} ; \quad B_2 = \frac{1}{2\Delta x}\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Mass - Spring System



$$m\ddot{\phi}(t) + K\phi(t) = u(t)$$

2-norm

if $f = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix} \in \mathbb{C}^n$

$$\|f\|_2^2 = f^* f = \bar{f}_1 \cdot f_1 + \bar{f}_2 \cdot f_2 + \cdots \bar{f}_n \cdot f_n$$

if $f \in L_2[-1,1]$

$$\|f\|_2^2 = \int_{-1}^{1} f^*(x) f(x)\, dx$$
