09-07-11

Distributed Systems

Fall '11, EE 8235

Terminology:

1) Spatially distributed systems : in addition to time, there is
(extended)
a spatial independent variable

2) Infinite dimensional systems
(Distributed parameter systems)
- partial differential equations (PDEs)
- Delay equations

3) Large-scale systems (interconnected systems)
Many degrees of freedom, High dynamical order.

First part of the course : (1) and (2)
Second " " " : (3)

Example

(1) Heat equation in 1D (one spatial direction)
(diffusion)

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial x^2}$$

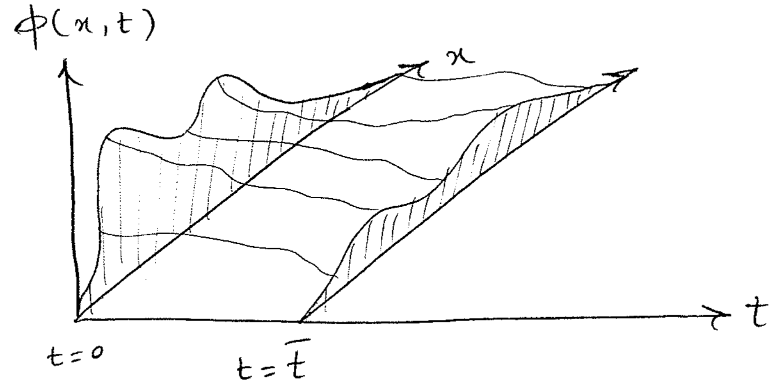
unforced problem (no input)

ϕ ... field of interest
(e.g. temperature distribution)

$\phi(x, t)$

$\left.\begin{array}{l} x \text{ ... spatial variable} \\ t \text{ ... time} \end{array}\right\}$ independent variables

A posible objective: find $\phi(x,t)$



x .... a continuous Spatial variable

e.g. $x \in [-1,1]$

Note: if x belongs to a finite interval, we will usually normalized it to $[-1,1]$

$$\bar{x} \in [0, L] \xrightarrow{\text{affine}} x \in [-1,1]$$

$$x = a\bar{x} + b$$

$$b = -1 \quad ; \quad a = \frac{2}{L}$$

$$\frac{\partial \phi}{\partial \bar{x}} = \frac{\partial x}{\partial \bar{x}} \frac{\partial \phi}{\partial x} = a \frac{\partial \phi}{\partial x}$$

Derivatives change after the transformation

Question what do we need to uniquely determine $\phi(x,t)$ from $\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial x^2}$?

Answer Need initial and boundary Conditions

I.C. : $\phi(x, t=0) = \phi_0(x)$

(an initial temperature distribution)

B.C. : Many possibilities, but need two B.C.

$\downarrow$

degree of $\frac{\partial}{\partial x}$

(a) $\phi(x = \pm 1, t) = 0$ (Dirichlet)

(b) $\frac{\partial \phi}{\partial x}(x = \pm 1, t) = 0$ (Neumann)

(c) Combination of (a) and (b)
(Linear)

For the case of forced Heat equation

$$\frac{\partial \phi}{\partial t}(x,t) = \frac{\partial^2}{\partial x^2}\phi(x,t) + u(x,t)$$

$\downarrow$

spatially and temporally distributed input

$\downarrow$

Can be disturbance or Control

$+$ I.C. $+$ B.C.

Boundary input :

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial x^2}$$

I.C. $\phi(x,0) = \phi_0(x)$

B.C. $\begin{cases} \phi(x=-1,t) = u(t) \\ \phi(x=1,t) = 0 \end{cases}$