1. Problem 1.2 (from the textbook Predictive Learning see http://www.VCtextbook.com)

2. Problem 1.4

3. Consider daily prices of Vanguard Total Stock Market ETF (symbol VTI) reflecting US stock market. The daily percentage change of VTI can be viewed as a random variable

\[ X(t) = \frac{Z(t) - Z(t-1)}{Z(t-1)} \times 100\% \]

where \( Z(t) \) denotes the daily closing prices of VTI and index \( t \) denotes current (trading) day.

We can obtain observations of \( X(t) \), based on daily closing prices of VTI (available from many financial websites, e.g. from http://finance.yahoo.com, click “get quotes” for symbol VTI, and then go to “historical prices”).

(i) Using historical data for \( Z(t) \), obtain observations of random variable \( X \) during year 2017. Then find the following statistical characteristics for random variable \( X \):

(a) the histogram of observed \( X \)-values, using the range (-2\%, +2\%), and the bin size 0.2\%.
(b) sample mean and standard deviation of \( X \)
(c) can the histogram in part (a) be closely approximated by normal distribution? If so, specify the parameters of this normal distribution (its mean and standard deviation).

(ii) Using the same historical data, calculate 4-days moving average (MA) of \( X(t) \), i.e. \( MA(t) = \frac{X(t) + X(t-1) + X(t-2) + X(t-3)}{4} \), for the same year 2017.
For this random variable \( MA(t) \), repeat (a) and (b) of part (i).

(iii) Compare the mean and standard deviations calculated in parts (i) and (ii) using historical data, and try to find analytical (theoretical) relationships:

(a) between the mean values of \( X \) and \( MA \)
(b) between standard deviations of \( X \) and \( MA \) (*)

(*) For part (iii), assume that observed \( X \) – values are statistically independent.