## Supplementary Information:

# Structure and electronic transport in graphene wrinkles 

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## 1. Maximum height estimate of the standing collapsed wrinkle.

To estimate the maximum height, we assume that the wrinkle has a fixed amount of material (i.e. no sliding of graphene along the surface), and that it adopts the minimum-energy morphology. We therefore need to compare energies of the different structures for a fixed amount of excess graphene in the wrinkle. First, we estimate energy of the folded wrinkle. As illustrated in Fig. S1, the structure of folded wrinkle consists of the right and left bulb-shaped curves with similar radiuses and a flat trilayer region. The right bulb we approximate by a pair of arcs, concave and convex. The two left bulbs are approximated by the arcs of the same angles, with radiuses being different by the van der Waals distance $h$ separating graphene layers. The trilayer region has length $\lambda$, and the bilayer has length $\lambda+\xi$. The base is approximated by arcs of angle $\pi / 2$ and radius $R_{\mathrm{b}}$. The energy of the folded wrinkle in this model is given by:

$$
\begin{align*}
& E_{f}=\frac{\kappa}{2}\left(\frac{\pi}{R_{b}}+\frac{\theta_{1}}{R_{1}}+\frac{\theta_{1}}{R_{1}+h}+\frac{\theta_{2}}{R_{2}}+\frac{\theta_{2}}{R_{2}+h}+\frac{\theta_{3}}{R_{3}}+\frac{\theta_{4}}{R_{4}}\right) \\
& -\beta\left(\theta_{1}\left(R_{1}+\frac{h}{2}\right)+\theta_{2}\left(R_{2}+\frac{h}{2}\right)+2 \lambda+\xi\right)  \tag{1}\\
& +\beta_{\text {sub }}\left(2 R_{b}+h\right)
\end{align*}
$$

where $\kappa$ is graphene bending stiffness and $\beta$ is van der Waals adhesion energy. The first term reflects the bending energy, the second term reflects adhesion energies of the bilayer and trilayer regions, and the last term reflects the adhesion energy cost to peel off
graphene from the substrate. We will use $\beta_{\text {sub }}=\beta$. The excess length is defined as the length of the graphene fold minus the length of the flat substrate and it is given by:

$$
\begin{equation*}
L=\pi R_{b}+\theta_{1}\left(2 R_{1}+h\right)+\theta_{2}\left(2 R_{2}+h\right)+2 \lambda+\xi+\theta_{3} R_{3}+\theta_{4} R_{4}-\left(2 R_{b}+h\right) \tag{2}
\end{equation*}
$$

where relationships between the angles and radiuses are determined by the geometric constraints, see Fig. S1: $\quad \xi=R_{4} \sin \theta_{4}-R_{3} \sin \theta_{3}, \quad \theta_{1}-\theta_{2}=\pi / 2$, $R_{1} \sin \theta_{1}+R_{b}=h+\left(R_{2}+h\right)\left(1-\cos \theta_{2}\right), \quad \theta_{3}-\theta_{4}=\pi, \quad R_{3}\left(1-\cos \theta_{3}\right)=h+R_{4}\left(1-\cos \theta_{4}\right)$. Minimization of energy in Eq. (1) with respect to the five variational parameters $R_{\mathrm{b}}, R_{1}$, $\theta_{1}, R_{3}, \theta_{3}$ for a fixed excess length $L$ from Eq. (2) gives the energy of the fold as a function of $L$.

Similarly, we can estimate the energy of the standing collapsed wrinkle geometry, shown in Fig. S2:

$$
\begin{equation*}
E_{\text {sc }}=\frac{\kappa}{2}\left(\frac{\pi}{R_{b}}+\frac{2 \theta_{1}}{R_{1}}+\frac{2 \theta_{2}}{R_{2}}\right)-\beta \lambda+\beta_{\text {sub }}\left(2 R_{b}+h\right) \tag{3}
\end{equation*}
$$

The excess length here is given by:

$$
\begin{equation*}
L=\pi R_{b}+2 \lambda+2 \theta_{1} R_{1}+2 \theta_{2} R_{2}-2 R_{b}-h \tag{4}
\end{equation*}
$$

where $\theta_{1}=\theta_{2}+\pi / 2, \quad R_{1} \sin \theta_{1}=h / 2+R_{2}\left(1-\cos \theta_{2}\right)$ are found from the geometrical constraints. Therefore, there are three variational parameters: $R_{\mathrm{b}}, R_{1}, \theta_{1}$ which minimize the energy in Eq. (3).

Commonly used values of $\kappa=1.4 \mathrm{eV}^{1}$ and $\beta$ corresponding to 40 meV adhesion energy per carbon atom $^{2,3}$ suggest an intrinsic length scale $R_{0}=\sqrt{\kappa / 2 \beta} \approx 6.8 \AA$. Numerical energy minimization from Eq. (1) and (3) using parameters $R_{0}=6.8 \AA$ and
$h=3.4 \AA$ leads to the values of the variational radiuses of the left and right bulbs in the folded wrinkle: $R_{1} \approx 6.5 \AA, R_{3} \approx 4.9 \AA$ (see Fig. S1 caption) to be in very good agreement with the values found from the DFT optimized geometry of 5-6 $\AA^{4}$. The minimum energy of the standing collapsed wrinkle from Eq. (3) as a function of $L$ is given as $\frac{E_{s c}}{\beta R_{0}} \approx 14.78-\frac{L}{2 R_{0}}$, while minimum energy of the folded wrinkle from Eq. (1) is given by $\frac{E_{f}}{\beta R_{0}} \approx 27.12-\frac{L}{R_{0}}$. The equal energy condition $E_{s c}=E_{f}$ is satisfied for $L_{m} \approx 24.7 R_{0}$, which defines a transition height from standing collapsed wrinkle to folded wrinkle as $R_{b}+\lambda+R_{1}\left(1-\cos \theta_{1}\right)+R_{2} \sin \theta_{2}$, where $\lambda \approx 7.9 R_{0}$ is found from Eq. (4). The height of the standing wrinkle at the transition (i.e. the maximum height) is about $12.4 R_{0} \approx 8.4 \mathrm{~nm}$, very close to $L_{m} / 2$.

## 2. Electrostatic modeling of the trilayered folds regions

We model the electrostatics of the graphene fold as a tri-layer graphene system, assuming that the graphene layers are electrically decoupled from one another. Through the Poisson equation, the Dirac point potential in each layer with respect to Fermi energy can be computed as follows,

$$
\begin{align*}
& V_{2}=-\frac{d_{0}}{\varepsilon_{0}} n_{3}+V_{3} \\
& V_{1}=-\frac{d_{0}}{\varepsilon_{0}}\left(n_{2}+n_{3}\right)+V_{2}  \tag{5}\\
& V_{g}=-\frac{1}{C_{g}}\left(n_{1}+n_{2}+n_{3}\right)+V_{1}
\end{align*}
$$

where $V_{3}$ is given a priori. $d_{0}=3.4 \AA$ is the graphene interlayer separation, $\varepsilon_{0}$ is the free space permittivity, $C_{g}$ is the back gate capacitance and $V_{g}$ is the applied gate bias. In Fig. 4a of the main manuscript, the calculated carrier densities assumed a finite electron-hole puddle densities $n_{0}=6.5 \times 10^{11} \mathrm{~cm}^{-2}$ estimated from Hall measurements. The fractional carrier population in the graphene layer closest to the gate, i.e. $n_{1} / n$ where $n=n_{1}+n_{2}+n_{3}$, is closer to unity at larger $V_{g}$. On the other hand, the layer densities are more equally
distributed when $V_{g}$ is biased near the Dirac point. This carrier redistribution within the trilayered graphene system is a consequence of nonlinear screening ${ }^{4}$, and is crucial to explaining our experimental observations.

## 3. Diffusive transport modeling along/across a graphene fold

We discuss first electronic transport along a graphene fold. The effective electrical conductivity $\sigma_{\text {eff }}$ in the diffusive limit can be written as,

$$
\begin{equation*}
\sigma_{e f f}=\frac{W_{f}}{W}\left(\sigma_{1}+\sigma_{2}+\sigma_{3}\right)+\frac{W-W_{f}}{W} \sigma \tag{6}
\end{equation*}
$$

where $\sigma_{j}$ refers to the electrical conductivity in the $j^{\text {th }}$ layer and $\sigma$ is the electrical conductivity in monolayer graphene i.e. control devices. $W_{f}$ is the width of the graphene fold, estimated from SEM to be $\approx 0.14 \mu \mathrm{~m}$, and $W$ is the device width. In addition, the electrical conductivity $\sigma$ as a function of the carrier density $n$ can be determined through Hall measurements. The carrier mean-free-path, $\lambda_{\text {MFP }}(n)$, can simply be derived from $\sigma=\frac{4 e^{2}}{\pi h} \sqrt{\pi n} \lambda_{M F P}{ }^{5}$. If each graphene layer in the fold also follows the same $\lambda_{M F P}(n)$ functional relationship, then the respective $\sigma_{j}$ are also known. In this case, the calculated $\sigma_{\text {eff }}$ is shown in Fig. 4c of the main manuscript, yielding good agreement. Electronic transport across a graphene fold can be modeled in similar fashion, with $\sigma_{\text {eff }}$ written as,

$$
\begin{equation*}
\frac{1}{\sigma_{e f f}}=\frac{L_{f}}{L}\left(\frac{1}{\sigma_{1}}+\frac{1}{\sigma_{2}}+\frac{1}{\sigma_{3}}\right)+\frac{L-L_{f}}{L} \frac{1}{\sigma_{0}} \tag{7}
\end{equation*}
$$

where $L_{f}$ is the length of the graphene fold, estimated from SEM to be $\approx 0.14 \mu \mathrm{~m}$, and $L$ is the device length.

## 4. Quantum transport modeling of standing collapsed wrinkle

Here we elaborate on the electronic transport calculation of the standing collapsed graphene wrinkle in the main manuscript. We assume that the transport direction is along the armchair direction, as illustrated in Fig. 5a. The Hamiltonian $H$ is described by a nearest neighbor $p_{z}$ tight-binding model ${ }^{6}$ including both in-plane and out-of-plane couplings,

$$
\begin{equation*}
H=\sum_{i} V_{i} a_{i}^{\dagger} a_{i}+\sum_{<i j>} t_{i j} a_{i}^{\dagger} a_{j}+\sum_{i j} s_{i j} a_{i}^{\dagger} a_{j} \tag{8}
\end{equation*}
$$

where $V_{i}$ denote the on-site energy, $t_{i j}$ the in-plane coupling and $s_{i j}$ the out-of-plane coupling. Explicitly, they are expressed as,

$$
\begin{align*}
& t_{i j}=\frac{v_{i j} \times p_{i}}{r_{i j}} \frac{v_{i j} \times p_{j}}{r_{i j}} \frac{\left(v_{i j} \times p_{i}\right) \cdot\left(v_{i j} \times p_{j}\right)}{\left|v_{i j} \times p_{i} \| v_{i j} \times p_{j}\right|} \varepsilon_{p p}^{\pi} \\
& s_{i j}=-\alpha \gamma \exp \left(-\frac{r_{i j}-r_{p}}{\delta}\right) \frac{p_{i} \cdot v_{i j}}{r_{i j}} \frac{p_{j} \cdot v_{i j}}{r_{i j}} \tag{9}
\end{align*}
$$

where $p_{i}$ refers to the local out-of-plane vector, $v_{i j}$ is the bond vector and $r_{i j}=\left|v_{i j}\right|$. Parameters $r_{p} \approx 0.34 n m$ refers to the equilibrium graphene interlayer separation, $\gamma \approx 0.119 \varepsilon_{p p}^{\pi}$ is the out-of-plane coupling energy, $\delta \approx 0.185 \times \sqrt{3} r_{0}$ where $r_{0}=0.142 \mathrm{~nm}$ is the carbon-carbon bond-length and $\alpha \approx 1.4$ is a fitting parameter ${ }^{6}$.

Electronic transport across the structure is calculated using the non-equilibrium green function method ${ }^{5}$ within the Landauer formalism, assuming periodic boundary condition along the transverse width direction. The transmission function $\mathrm{T}\left(k_{y}, E\right)$ can then be calculated. The finite temperature device conductance can be calculated using,

$$
\begin{equation*}
G=\frac{2 e^{2}}{h} \int_{-\infty}^{\infty} \frac{1}{k_{B} T} \exp \left[\frac{E-\mu}{k_{B} T}\right] f^{2}(E)\left[\sum_{k_{y}} \mathrm{~T}\left(k_{y}, E\right)\right] d E \tag{10}
\end{equation*}
$$

where $f(E)$ is the Fermi Dirac distribution. The resistance associated with the standing collapsed wrinkle can then be calculated after subtracting off the quantum contact resistance. In our calculations, we assume that the electrostatic doping of the flat region to be 0.2 eV , and undoped in regions which are raised, namely the collapsed bilayer and the structure subtended from it. Temperature is taken to be 300 K as per experiments.

## 5. Temperature dependence of conductivity

The conductivities of the graphene device at 4.2 K and 300 K are shown in Fig. S3. We observe that the conductivity is nearly unchanged when the temperature is decreased from 300 K to 4.2 K .

## 6. Measurements of Hall Mobility

We performed standard Hall measurement to obtain the resistivity tensor and then the conductivities $\sigma_{x x}$ and $\sigma_{x y}$, from which the carrier mobility $\mu$ and carrier density $n$ can be extracted.

$$
\begin{align*}
& \mu=\frac{1}{B} \frac{\sigma_{x y}}{\sigma_{x x}}  \tag{11}\\
& n=\frac{\sigma_{x x}\left(1+\mu^{2} B^{2}\right)}{\mu q} \tag{12}
\end{align*}
$$

where B is the magnetic field. These quantities are plotted in Fig. S4, obtained at 300K. We emphasize that the extraction method breaks down when the graphene is biased near the Dirac point, the range highlighted in the plot. The observed downturn in the mobility is unphysical, an artifact of the extraction method which ignores the two carrier nature of transport near the Dirac point ${ }^{7}$. Outside this region, the measured mobility clearly shows a decreasing mobility with increasing doping.

## 7. Dirac point shifts due to folds

The statistical sampling of Dirac voltage for the graphene Hall-bars with and without fold is shown in Fig.S5: (a) across fold vs no fold; (b) along the fold vs no fold. The statistics of Dirac voltage value indicate that the presence of a fold does not lead to significant changes in the Dirac point shifts, hence of the doping level. This indicates that most of the trapped impurities reside in the substrate or the $\mathrm{SiO}_{2}$-graphene interface.

## Reference:

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Fig. S1. Schematics of the folded graphene wrinkle. The minimum energy from Eq. (1) corresponds to the values of the variational parameters $R_{b}=R_{0}, R_{1} \approx 0.950 R_{0}$, $R_{2} \approx 3.025 R_{0}, \theta_{1} \approx 0.764 \pi, \theta_{2} \approx 0.264 \pi, R_{3} \approx 0.714 R_{0}, R_{4} \approx 2.555 R_{0}, \theta_{3} \approx 1.246 \pi$, $\theta_{4} \approx 0.246 \pi$, where $R_{0}=6.8 \AA$.


Fig. S2. Schematics of the standing collapsed graphene wrinkle. The minimum energy from Eq. (3) corresponds to the values of the variational parameters $R_{b}=\sqrt{2 \pi /(2+\pi)} R_{0}$, $R_{1} \approx 0.967 R_{0}, R_{2} \approx 3.401 R_{0}, \theta_{1} \approx 0.685 \pi, \theta_{2} \approx 0.185 \pi$, where $R_{0}=6.8 \AA$.


Fig.S3. Conductivity as a function of $\mathrm{V}_{\mathrm{BG}}-\mathrm{V}_{\text {Dirac }}$ at 4.2 K and 300 K for a graphene device (a) with no fold, (b) measured across the fold and (c) along the fold.


Fig.S4. Extracted (a) Hall mobility and (b) carrier density in graphene at 4 K , via the standard Hall measurement procedure.


Fig. S5. Statistics of Dirac voltage of graphene Hall-bars (a) across fold vs no fold, and (b) along fold vs no fold. The statistics are based on the data from 42 devices.

