Supplementary information: Anomalous Reflection Phase of Graphene Plasmons and its Influence on Resonators

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I. HOMOGENEOUSLY DOPED GRAPHENE

We will solve the problem of the graphene plasmon (GP) reflection from the edge by assuming that a single mode (GP) from the region $x < 0$ couples to the "electromagnetic continuum" in the region $x > 0$.

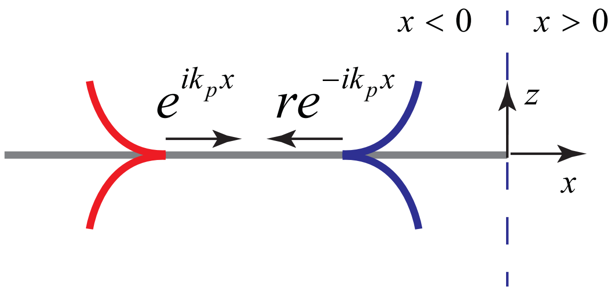

FIG. 1: The schematic of the studied system: the incident GP impinge onto the graphene termination at $x = 0$ and generates the reflected GP

For simplicity and symmetry we assume that graphene is free-standing. Let us represent the magnetic field of the incident GP in the region $x < 0$ as follows (we use the notation $H \equiv H_y$)

$$H_i(x,z) = \pm e^{ik_p x \pm ik_{pz} z}, \tag{1}$$

where $+$ and $-$ correspond to the regions $z > 0$ and $z < 0$ respectively and

$$k_p = g\sqrt{1 - 1/\alpha^2}, \quad k_{pz} = -g/\alpha, \tag{2}$$

with $g = \omega/c$ being the free-space wavevector and $\alpha = 2\pi\sigma/c$ being the normalized conductivity of graphene. Using the Maxwell equations, z-component of the electric field reads

$$E_{zi}(x,z) = \mp \frac{k_p}{g} e^{ik_p x \pm ik_{pz} z}. \tag{3}$$

We will seek for the reflected GP in the following form:

$$\begin{aligned} H_r(x,z) &= \pm r e^{-ik_p x \pm ik_{pz} z}, \\ E_{zr}(x,z) &= \pm \frac{k_p}{g} r e^{-ik_p x \pm ik_{pz} z}, \end{aligned} \tag{4}$$

where we have introduce the reflection coefficient r. Then the total fields in the region $x < 0$ read

$$\begin{aligned} &H_<(x,z) = H_i(x,z) + H_r(x,z) = \\ &\pm (e^{ik_p x} + r e^{-ik_p x}) e^{\pm ik_{pz} z}, \\ &E_{z<}(x,z) = E_{zi}(x,z) + E_{zr}(x,z) = \\ &\mp (e^{ik_p x} - r e^{-ik_p x}) \frac{k_p}{g} e^{\pm ik_{pz} z}. \end{aligned} \tag{5}$$

Now let us represent the field in the region $x > 0$ in the form of the plane waves expansion

$$\begin{aligned} H_>(x,z) &= \int_{-\infty}^{\infty} dk_z h(k_z) e^{ik_x x + ik_z z}, \\ E_{z>}(x,z) &= -\int_{-\infty}^{\infty} dk_z h(k_z) \frac{k_x}{g} e^{ik_x x + ik_z z}, \end{aligned} \tag{6}$$

where $k_x = k_x(k_z) = \sqrt{g^2 - k_z^2}$

The boundary conditions along the line $x = 0$ read

$$\begin{aligned} H_<(0,z) &= H_>(0,z), \\ E_{z<}(0,z) &= E_{z>}(0,z). \end{aligned} \tag{7}$$

Using (6) and (5) we have

$$\begin{aligned} (1+r)\psi(z) &= \int_{-\infty}^{\infty} dk_z h(k_z) e^{ik_z z}, \\ (1-r)\frac{k_p}{g}\psi(z) &= \int_{-\infty}^{\infty} dk_z h(k_z) \frac{k_x}{g} e^{ik_z z}, \end{aligned} \tag{8}$$

where $\psi(z)$ is the GP profile function

$$\psi(z) = \pm e^{\pm ik_{pz} z}. \tag{9}$$

At $x = 0$ the z-component of the electric field must be continuous, so we project the second line of (9) by $\frac{e^{-ik_z z}}{2\pi}$. However, the magnetic field does not need to be continuous, due to the existence of surface currents, so we project the first line of (9) by $\psi(z)$. After integration, we obtain:

$$\begin{aligned} &h(k_z)\frac{k_x}{g} = (1-r)\frac{k_p}{g}\frac{1}{2\pi}\int_{-\infty}^{\infty} dz \psi(z) e^{-ik_z z}, \\ &(1+r)\int_{-\infty}^{\infty} dz \psi^2(z) = \int_{-\infty}^{\infty} dk_z dz h(k_z)\psi(z) e^{ik_z z}, \end{aligned} \tag{10}$$

Substituting $h(k_z)$ from the first line into the second one, we find the linear equation for r:

$$(1+r)\int_{-\infty}^{\infty} dz\psi^2(z) =$$
$$(1-r)\frac{k_p}{g}\frac{1}{2\pi}\int_{-\infty}^{\infty} dk_z dz dz' \frac{g}{k_x}\psi(z)\psi(z')e^{ik_z(z-z')}. \tag{11}$$

The integrals in z, z' are trivially performed

$$\int_{-\infty}^{\infty} dz\psi^2(z) = \frac{1}{|k_{zp}|},$$
$$\int_{-\infty}^{\infty} dz dz' \psi(z)\psi(z')e^{ik_z(z-z')} = \frac{4k_z^2}{(|k_{zp}|^2+k_z^2)^2}, \tag{12}$$

Then (11) becomes

$$(1+r)\frac{1}{|k_{zp}|} = (1-r)\frac{2k_p}{\pi}\int_{-\infty}^{\infty} dk_z \frac{k_z^2}{k_x(|k_{zp}|^2+k_z^2)^2}. \tag{13}$$

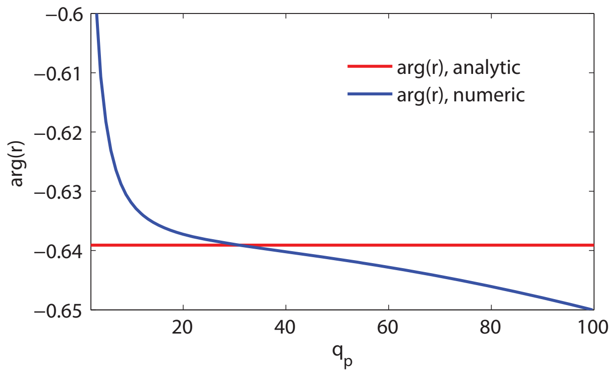

FIG. 2: The calculated phase of the reflection coefficient for GP at different values of momentum q_p. Both analytic (red line) and numeric (blue curve) results are shown.

From here we find the reflection coefficient explicitly

$$r = \frac{a-1}{a+1}, \tag{14}$$

where

$$a = \frac{2|q_{zp}|q_p}{\pi}\int_{-\infty}^{\infty} dq_z \frac{q_z^2}{q_x(|q_{zp}|^2+q_z^2)^2}. \tag{15}$$

In the last expression the dimensionless components of the wavevectors read

$$q_{zp} = -\frac{1}{\alpha}, \quad q_p = \sqrt{1-\frac{1}{\alpha^2}}, \quad q_x = \sqrt{1-q_z^2}. \tag{16}$$

The results of the numeric calculations for the reflection coefficient phase as a function of the GP momentum are shown in Fig.2 (blue curve). The integral in (15) has been calculated by using Cauchy theorem to avoid the branch point singularity $q_x = 0$ ($q_z = 1$).

The integral (15) can be strongly simplified if we take into account that the major contribution comes from large wavevectors, i.e from those with $q_z \gg 1$. In this case q in the denominator of the integrand can be replaced by $q \simeq iq_z$. Then the integration is analytic and the result reads simply

$$a = -2i/\pi, \tag{17}$$

so that the result is independent upon the GP wavevector and thus properties of graphene. According to Eqs. (17),(14) the reflection phase reads

$$\arg(r) = \arctan(-4\pi/(4+\pi^2)) \approx -0.64\pi. \tag{18}$$

This result is shown in Fig. 2 (red line) and is in an excellent agreement with the numerical simulations.

II. INHOMOGENEOUSLY DOPED GRAPHENE

A. Local reflection coefficient

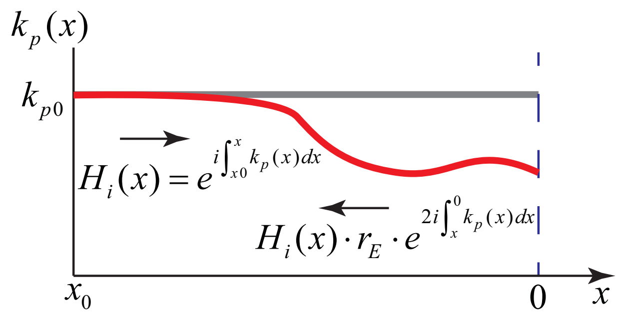

FIG. 3: The schematic of the geometrical optics approximation for graphene plasmon reflection from the edge in the inhomogeneously doped graphene.

Let us assume now that the conductivity of graphene is a function of distance, $\sigma = \sigma(x)$, see Fig. 3. Then if the variation is smooth enough, the plasmon propagation can be described by the geometrical optics. Namely, introducing the local plasmon wavevector $k_p(x) \propto 1/\sigma(x)$, one can represent the magnetic field of the incident plasmon propagating towards the edge as follows (we will omit the z-dependency assuming that we are staying infinitesimally close to the graphene face, for example at $z = 0^+$):

$$H_i(x) = e^{i\int_{x_0}^{x} dx' k_p(x')}, \tag{19}$$

where x_0 is a point where the phase of the magnetic field is zero. The incident plasmon propagates toward the edge (located at the position $x = 0$), experiences the reflection with the reflection coefficient r_E and propagates back. Then taking into account the progressive phase accumulation, the resulting reflected plasmonic field $H_r(x)$ can be written as

$$H_r(x) = H_i(x) r_E e^{2i\int_x^0 dx' k_p(x')}. \qquad (20)$$

The total magnetic field $H_{tot}(x)$ presents the sum of the incident and reflected fields $H_{tot}(x) = H_i(x) + H_r(x)$, so that

$$H_{tot}(x) = e^{i\int_{x_0}^x dx' k_p(x')} + r_E e^{i\int_{x_0}^x dx' k_p(x') + 2i\int_x^0 dx' k_p(x')}. \qquad (21)$$

Let us rewrite this equation in another useful form:

$$H_{tot}(x) = e^{i\varphi_0 + i\Delta\varphi_i(x)} \left[e^{ik_{p0}x} + r(x) e^{-ik_{p0}x} \right], \qquad (22)$$

where we have introduced the initial phase $\varphi_0 = -k_{p0}x_0$, the phase incursion for the incident field

$$\Delta\varphi_i(x) = \int_{x_0}^x dx' \Delta k_p(x') \qquad (23)$$

with $\Delta k_p(x) = k_p(x) - k_{p0}$; and a local reflection coefficient

$$r(x) = r_E e^{i\Delta\varphi(x)}, \quad \Delta\varphi(x) = 2i\int_x^0 dx' \Delta k_p(x'). \qquad (24)$$

Once the total field is known (for instance from the full wave simulations), both the local reflection coefficient $r(x)$ and reflection coefficient for the edge r_E can be extracted from Eqs. (22)-(24).

B. The shape of the conductivity profile

For the calculations of the reflection phase in case of the inhomogeneous profile of the conductivity, we will use the inverse square root type dependency in the vicinity of the edge $\propto 1/\sqrt{x}$. In order to avoid the singularity at $x = 0$, we displace the singular point to positive values of x, so that the profile has the following form

$$\sigma(x) = \sigma_0 F(x), \qquad (25)$$

where σ_0 is the conductivity far away from the perturbation (or "background" conductivity), $\sigma_0 = \sigma(-\infty)$ under the assumption that $F(-\infty) = 1$. The function $F(x)$ reads

$$F(x) = \begin{cases} 1, & x < x_0, \\ \frac{f(x)}{f(x_0)}, & x_0 \le x \le 0, \\ 0, & x > 0, \end{cases} \qquad (26)$$

with

$$f(x) = 1 + \frac{a}{\sqrt{|x - \delta|}}, \qquad (27)$$

where δ and a are the parameters controlling the maximal value of the conductivity at the position $x = 0$ (at the edge) and the growth, while x_0 presents the distance from the edge where the conductivity saturates to the constant value. In the calculations for the paper we take $x_0 = -1\,\mu$m and $\delta = 10$nm.

Notice that we do not follow any special model for the shape of the conductivity, but take the above profile for the proof of principle.

III. THE RIBBON-RIBBON INTERACTION

Fig. 4 shows the effect of the distance between the ribbons in the array. One can clearly see that for a larger separation (period) L the lowest-frequency resonance ($n = 0$) in the relative transmission is much closer to the curve predicted by a model (the white line).

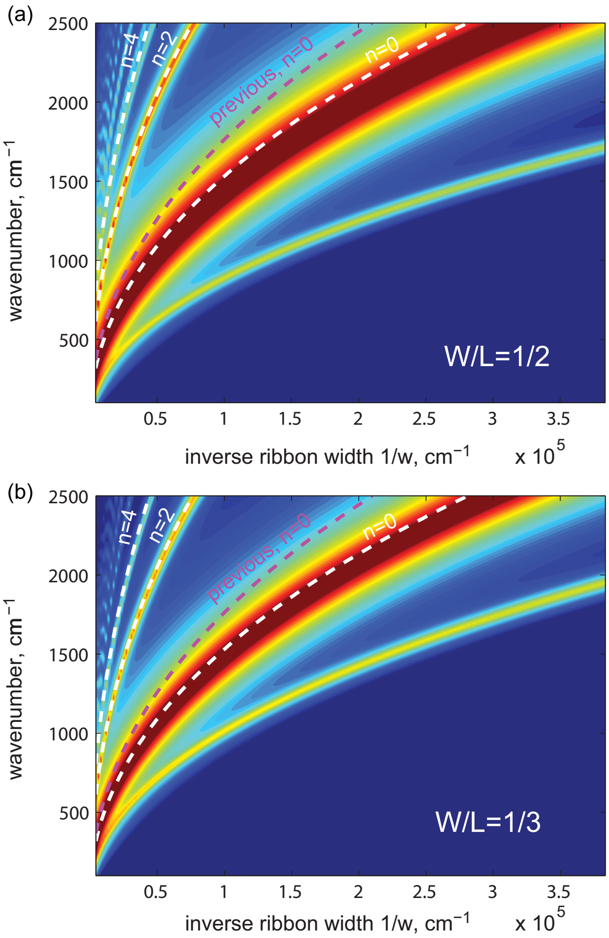

FIG. 4: Relative transmission $\delta T = |1 - T/T_0|$ (with respect to the one for the graphene-free substrate) through the array of free-standing graphene ribbons as a function of the inverse ribbon width and wavenumber. The discontinuous curves numbered by "0,2,..." correspond to the modes given by the model (Eq. (6) in the manuscript). The discontinuous curve marked as "previous, n=0" corresponds to $\Phi_R = -\pi$ and $n = 0$. The parameters used for graphene conductivity: temperature 300 K, Fermi level 0.55 eV, relaxation time of the charge carriers is 0.1 ps. (a) The period equal to twice the ribbon width, $L = 2W$. (b) $L = 3W$.