Beam deflectometry for measuring two-dimensional refractive index profiles in rectangular gradient-index (GRIN) optical materials

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Abstract: We present a numerical method for calculating two-dimensional index fields from measured boundary values of ray position and slope. Refractive index errors of <1% (RMS) of the total index range ($n_{max} - n_{min}$) are achieved using this approach.
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1. Introduction

GRIN materials belong to a class of inhomogeneous optical media whose refractive index varies with position. Accurate knowledge of the index field of a GRIN element is of paramount importance for its integration into optical systems. In recent studies, deflectometry principles have been utilized in interrogating weakly refracting index fields in the x-ray regime [1,2]. The approximations associated with x-ray measurements become invalid at longer wavelengths (e.g. visible) where the index of materials is significantly larger than unity. In a previous study, we showed that boundary measurements of ray position and slope can be bootstrapped to ascertain the index profile of a thick one-dimensional (1-D) GRIN element, provided the index is known at some location [3]. In this study, we propose a method for measuring the two-dimensional (2-D) refractive index of rectangular GRIN elements using boundary measurements of ray position and ray slope. The method can be generalized to three-dimensional (3-D) GRIN elements as well.

2. Linear algebraic system formulation

Where diffraction is negligible, the ray equation of geometric optics governs the propagation of light inside a GRIN medium:

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = \nabla n \quad , \tag{1}$$

where ds is the arc length along the ray trajectory, $\vec{r}$ is the position of a point on the trajectory and n is the refractive index. Using small angle approximations, it is straight-forward to show from Eqn. (1) that the normal component of the local index gradient induces a change in the ray's direction and the overall angular deflection of an interrogation ray can be expressed as a projection of the partial derivatives of the index field:

$$\Delta\theta = \int_a^b ds \cdot \vec{\nabla} w \cdot \hat{t} = \int_a^b ds \cdot \left[-sin(\phi)\frac{\partial w}{\partial x} + cos(\phi)\frac{\partial w}{\partial y}\right] \quad , \tag{2}$$

where $\hat{t} = -sin(\phi)\hat{\imath} + cos(\phi)\hat{\jmath}$ denotes the normal unit vector to the ray trajectory and $\vec{\nabla} w = \frac{\partial w}{\partial x}\hat{\imath} + \frac{\partial w}{\partial y}\hat{\jmath}$ is the gradient of the logarithmic index field $w = ln[n]$. Because $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ are the partial derivatives of an underlying potential function w, they must satisfy

$$\vec{\nabla} \times \vec{\nabla} w = \frac{\partial}{\partial y}\frac{\partial w}{\partial x} - \frac{\partial}{\partial x}\frac{\partial w}{\partial y} = 0 \quad . \tag{3}$$

A rectangular grid of discrete sample points is used to represent the partial derivatives in our approach. The values of each field quantity at these sample points are treated as the degrees of freedom (DoFs) in the system. A set of simultaneous algebraic equations in the DoFs of the system is constructed from Eqn. (2) and Eqn. (3). In order to linearize the system, ds and ϕ in Eqn. (2) are assumed to be independent of $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ and approximate ray trajectories are generated from measured boundary values of ray position and slope. The linear system is subsequently inverted using LSQR [4] and the computed partial derivatives are integrated to reconstruct the index field. Assuming knowledge of the integration constant (can be obtained from boundary value measurements of the index field), the reconstructed index field is used to make corrections to approximate trajectories. A new system is generated from updated trajectories and used to improve the reconstruction. This process is repeated until the ray trajectories and the index field are consistent according to Eqn. (1).

3. Numerical simulation

In theory, the minimum number of measured projections needed to invert the system is the number of DoFs. In practice, redundant measurements are typically needed to suppress error contributions from individual measurements. In our simulation, numerical ray tracing is used to generate boundary values for ray position and ray slope for interrogations rays in the rectangular index profile shown in Fig. $1(a)$. 225 interrogations rays propagating from x_{min} to x_{max} and another 225 rays propagating from y_{min} to y_{max} are used to invert the system. The index field is reconstructed on an 11×11 grid following the procedure outlined in the previous section. Reconstruction error in the index is shown in Fig. $1(b)$, with an RMS value of 1.7×10^{-3} refractive index units (R.I.U.).

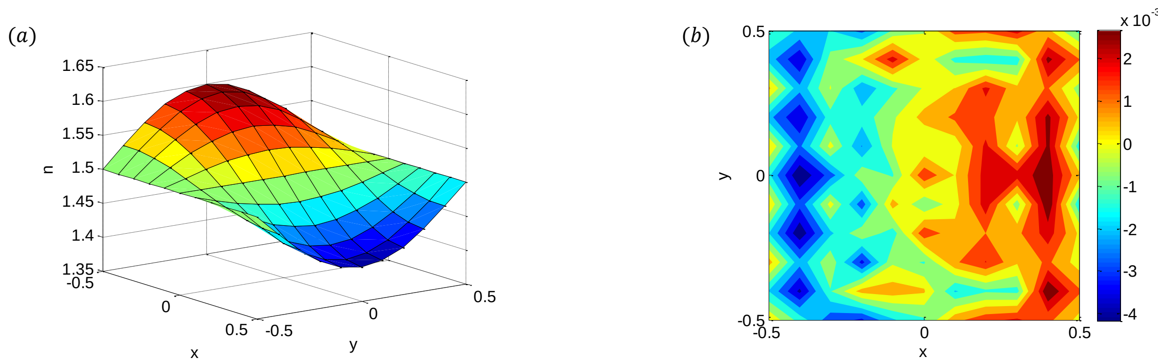

Fig. 1. (a) Test index profile used in simulation and (b) reconstruction error in refractive index.

4. References

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