The Physical Origin of Kramers-Kronig Self-Phasing in Coherent Laser Beam Combination

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Abstract: The Kramers-Kronig self-phasing observed in coherently coupled fiber laser arrays is experimentally shown to originate from a change in the supermode intensity distribution. The conditions that lead to accurate self-phasing are modeled and experimentally confirmed.

OCIS codes: 140.3298, 140.3410

Coherent beam combining has proven to be an effective method of increasing the radiance of a laser system. Over the past several years, research groups have successfully applied both active [1] and passive [2] coherent beam combining architectures to fiber lasers. Active beam combining employs sensors and actuators to correct for phase errors introduced by the environment. In contrast, passive beam combining relies on various physical effects to correct these same phase errors. Recently, we have developed techniques to isolate individual self-phasing mechanisms in order to study their physical origin in detail [3]. In particular, the Kramers-Kronig effect has been studied in the absence of other phase-inducing mechanisms and has been shown to play an important role in the self-phasing of fiber laser arrays.

The key to a careful examination of Kramers-Kronig self-phasing is to eliminate all other self-phasing mechanisms. We accomplish this by using a carefully designed and fabricated dual-core fiber that is path-length balanced to a few tens of micrometers. Light from the two fiber cores is then coherently coupled by a Dammann grating so that the optical paths of each beam are nearly identical. This optical path length balancing prevents wavelength shifting, thermal expansion, and thermally induced index changes from effecting the differential phase between the two laser cores. The experimental set-up is shown in fig. 1a, where the fiber is square cleaved on the left end and angle-cleaved on the right end. Light from the right side of the fiber is combined by a Dammann grating before being passed through a polarizer and reflecting from a littrow grating. This cavity can be interrogated by a tunable probe laser introduced via a beam splitter on the right side of the cavity. This probe laser measures the total phase difference between the two laser channels, including the phase imparted by the grating and any phase shifts introduced by the two cores of the fiber laser. Fig. 1b is a reproduction of a previously published results showing the self-phasing effect. The applied phase shown along the x-axis was introduced by shifting the grating perpendicular to the grating lines. The measured total phase, shown in along the y-axis, clearly shows the fiber compensating for the applied phase in such a manner as to keep the total cavity operating at the low-loss phase states of kλ. In the present paper, we explore the physical mechanism that produces this remarkable self-phasing effect.

The first clue of the origin of the Kramers-Kronig phase shift comes from measuring the intensity distribution of the supermode as a function of applied phase error. We observed that as the applied phase error increases, the intensity of one core increases while the other core decreases. An independent measurement of the fiber gain saturation curve

Fig. 1. (a) Experimental set-up; (b) Measurements of laser output power and total path length errors as a function of applied path length error [3].
allowed us to convert these intensity variations into gain variations, thus producing a difference in gain between cores. Next, we measured the KK phase as a function of gain. This relationship is seen to be linear, with a corresponding alpha parameter of approximately 7. By using this relationship, we could calculate the KK phase induced by the intensity change in the supermode. This calculated phase shift is plotted in fig. 2a along with the phase shift measured interferometrically from the cavity. It is clear that the correspondence is quite close, indicating that the supermode intensity change, mediated by the gain saturation and the KK phase shift, is ultimately responsible for the self-phasing behavior.

![Graphs showing phase shift](image)

**Fig. 2.** a) Measurement of the induced KK phase shift (black) as a function of applied phase error, compared with calculated KK phase shifts based on supermode intensity measurements, gain saturation measurements, and the KK response of the fiber. b) Result of a nonlinear cavity analysis.

The question remains why the supermode intensity distribution changes, and in particular why the change is able to effectively correct for the applied cavity phase error. To answer this question, we modelled the round-trip propagation of light in the cavity as consisting of two parts. The first part consisted of a simple coupling matrix that describes a Dammann grating [4]. The second part described the propagation of light through the fibers, and contained the Kramers-Kronig effect. Thus, a specific supermode would specify a given circulating intensity distribution, which in turn would change the gain through the gain saturation equation. This differential gain would then change the differential phase of the cores, which finally would change the calculated supermode. By iterating until a stable solution was found, we were able to calculate a specific supermode intensity and corresponding total phase shift for each applied phase shift. The total calculated cavity phase shift is shown in fig. 2b above. The figure can be seen to have the characteristic stair-case shape observed in fig. 1a, indicating efficient self-phasing. We thus believe this explains the origin of the self-phasing effect. We note that the phase response does not always have the desired shape shown in fig. 2b. The array must contain a small difference in coupling efficiency between the two cores, and the alpha parameter must be within a particular range. Alpha parameters that were chosen outside this range were seen to give rise to chaotic responses.

In conclusion, we have performed precise measurements on a highly controlled coherently coupled laser cavity and have determined the origin of the self-phasing behavior. A thorough understanding of the self-phasing effects of fibers may lead to designs that exploit this phasing mechanism, and may potentially improve future passive and active coherently combined cavities.

**References**


