Coupled Oscillator base Computing: Using Nature to Solve Difficult Problems

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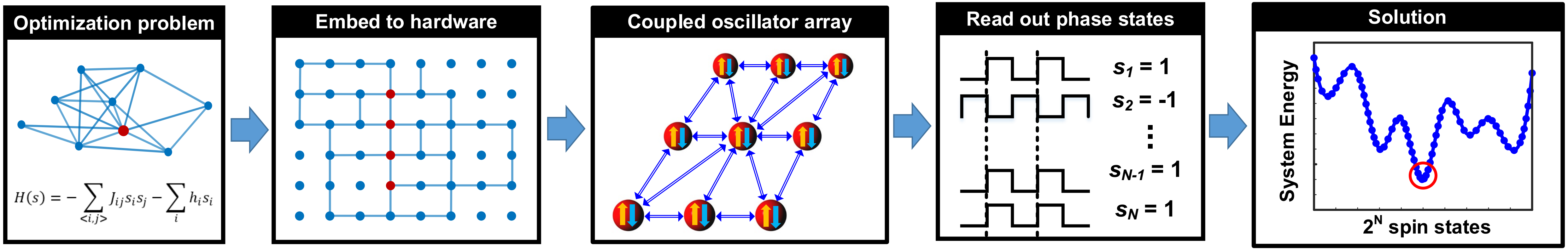
Outline

- Introduction to Ising Computers
- Case Study: 560 Coupled Oscillator Test Chip
- Summary

Ising Spin Glass Model

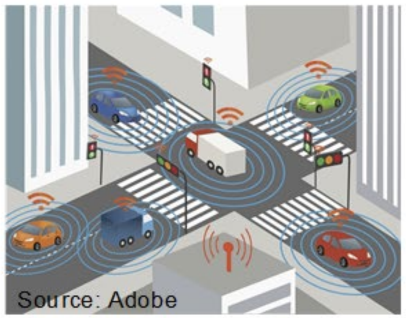


- A promising approach for efficiently solving NP-hard or NP-complete problems (e.g. combinatorial optimization problems, Boltzman machines, associative memories, Karp's 21 NP-complete problems)

$H(s) = -\sum_{<i,j>} J_{ij} s_i s_j - \sum_i h_i s_i$: Ising Hamitonian (Cost Function)

s_i, s_j : Spin state {+1 or -1} J_{ij} : Coupling strength h_i : local field strength

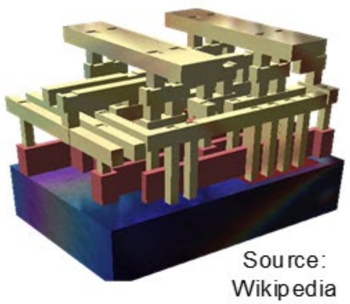
Autonomous vehicles



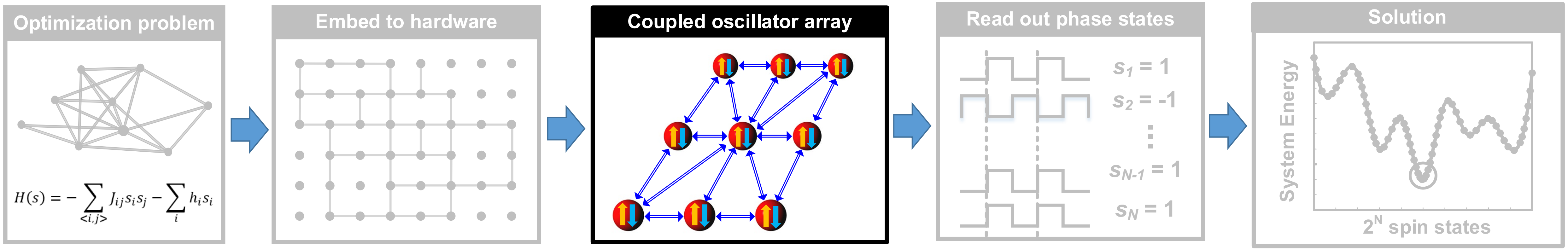
Communication networks

Smart grid

VLSI routing



Using Nature to Find the Ground State



Random states (time=0)

Same states (time = 1 min)

Youtube: Coupled Metronomes

Example Problem #1: Factorizing 15

$$\mathrm{p} = (x_1\ 1)_2\ , \mathrm{q} = (x_2\ x_3\ 1)_2$$

$$H = (15 - pq)^2$$

$$H = 128x_1x_2x_3 - 56x_1x_2 - 48x_1x_3 + 16x_2x_3 - 52x_1 - 52x_2 - 96x_3 + 196$$

$$H_{mod} = 200x_1x_2 - 48x_1x_3 - 512x_1x_4 + 16x_2x_3 - 512x_2x_4 + 128x_3x_4 - 52x_1 - 52x_2 - 96x_3 + 768x_4 + 196$$

S. Jiang, et al., “Quantum Annealing for Prime Factorization”, Scientific Reports 2018

Example Problem #2: Graph Coloring

For graph $G(V, E)$ of the map problem—no two vertices, V, connected by an edge, E, should select the same color from set C—construct a cost function with binary variables, $x_{v,c} = 1$ when $v \in V$ selects color $c \in C$, by implementing two constraints:

$$(\sum_c x_{v,c} - 1)^2,$$

which has minimum energy (zero) when vertices select one color only, and

$$\sum_c \sum_{v_a, v_b \in E} x_{v_a,c} x_{v_b,c},$$

which adds a penalty if the vertices of an edge select the same color.

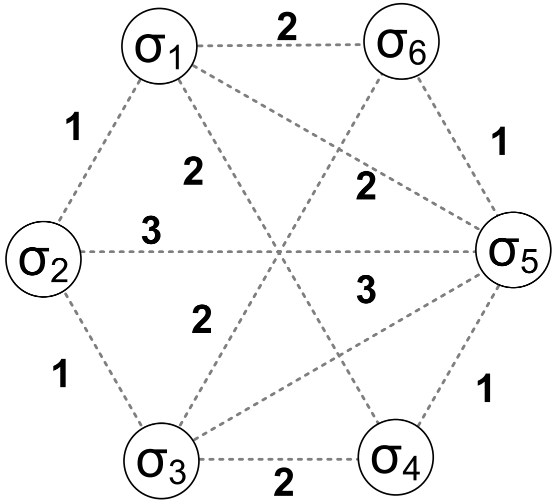
These constraints give a QUBO,

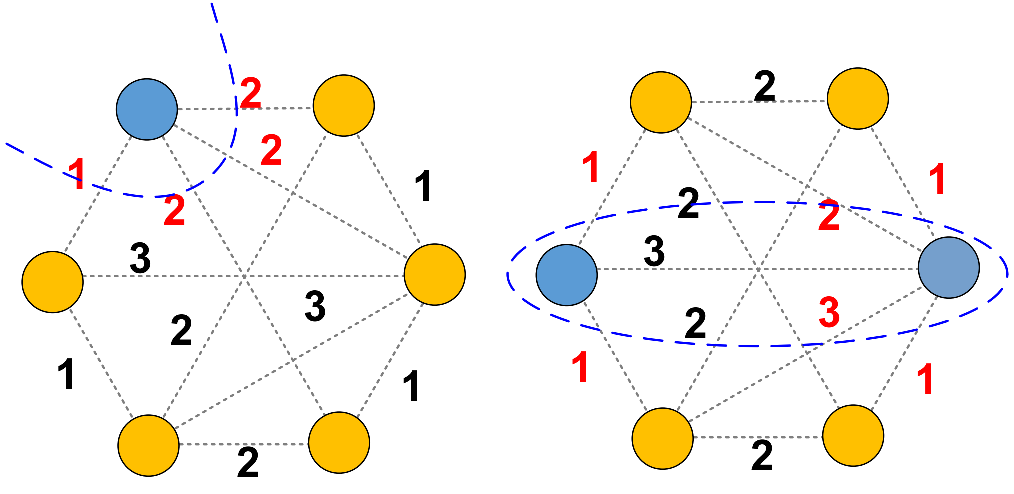
$$E(x_v, x_{v_a,v_b}) = \sum_v (\sum_c x_{v,c} - 1)^2 + \sum_c \sum_{v_a, v_b \in E} x_{v_a,c} x_{v_b,c}.$$

e.g. $x_{Minn,Red} = 0, x_{Minn,Blue} = 0,$
$x_{Minn,Sand} = 1, x_{Minn,Green} = 0$
$x_{Wisc,Red} = 0, x_{Wisc,Blue} = 1,$
$x_{Wisc,Sand} = 0, x_{Wisc,Green} = 0$

D-Wave Problem-Solving Handbook, 8/13/20

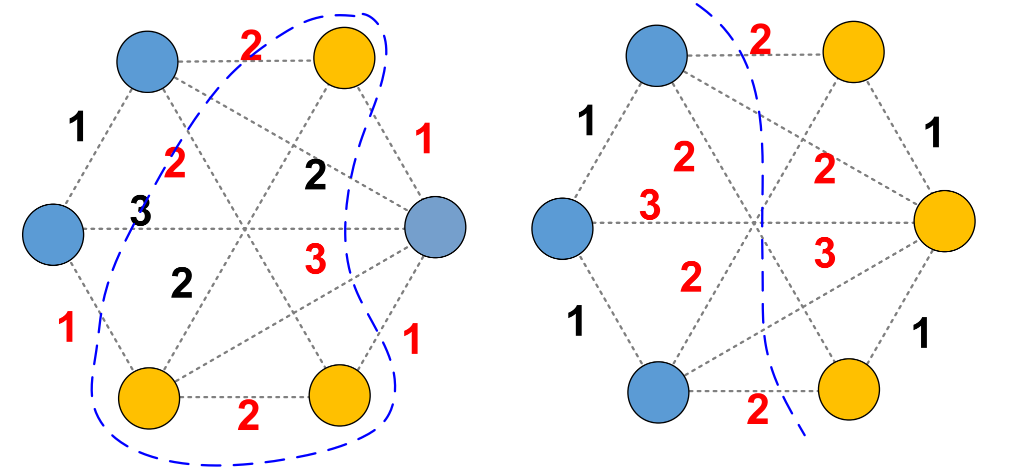
Example Problem #3: Finding Max-cut





Cut size = 7

Cut size = 9

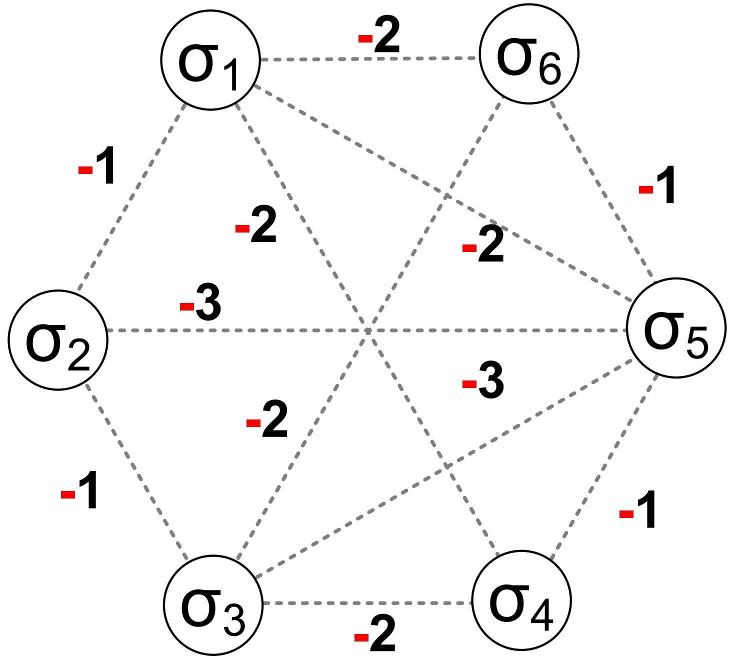


Cut size = 12

Cut size = 16 (max)

- The problem of finding a maximum cut in a graph is known as the Max-Cut Problem
- Finding max-cut of a graph is an *NP-hard* problem

Example Problem #3: Finding Max-cut



$$H(\sigma) = -\sum_{i,j} (-w_{ij})\,\sigma_i\sigma_j$$
$$= \sum_{diff\ group} (-w_{ij}) + \sum_{same\ group} w_{ij}$$
$$= \sum_{diff\ group} (-w_{ij}) + \left[\sum_{i,j} w_{ij} - \sum_{diff\ group} w_{ij}\right]$$
$$= \sum_{all} w_{ij} - 2 \times \sum_{diff\ group} w_{ij}$$

Cut size

H = Hamiltonian of the system
σ_i = Spin status of magnet i {+1 or -1}
w_{ij} = weight between magnets i and j

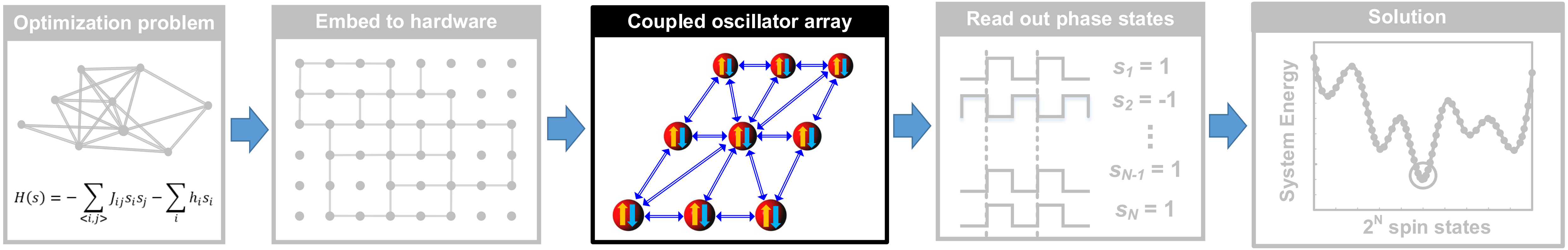
- Ising Hamiltonian = [sum of all weights] – 2×[cut size]

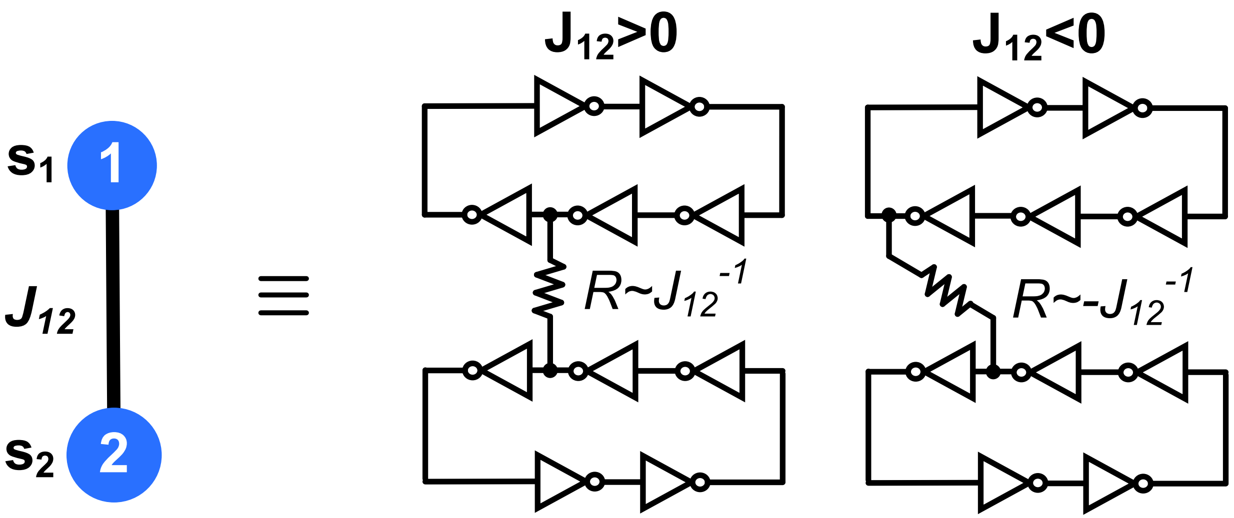
Other NP Problems Mappable to the Ising Model

- Partitioning problems (e.g. max cut)
- Binary integer linear programming
- Covering and packing problems
- Problems with inequalities
- Coloring problems (e.g. graph coloring)
- Hamiltonian cycles (e.g. traveling salesman)
- Tree problems
- Graph isomorphisms
- …

A. Lucas, "Ising formulations of many NP problems", Frontiers in Physics, Feb. 2014

Using Coupled Oscillators to Find the Ground State





$$H(s) = -J_{ij} s_i s_j$$

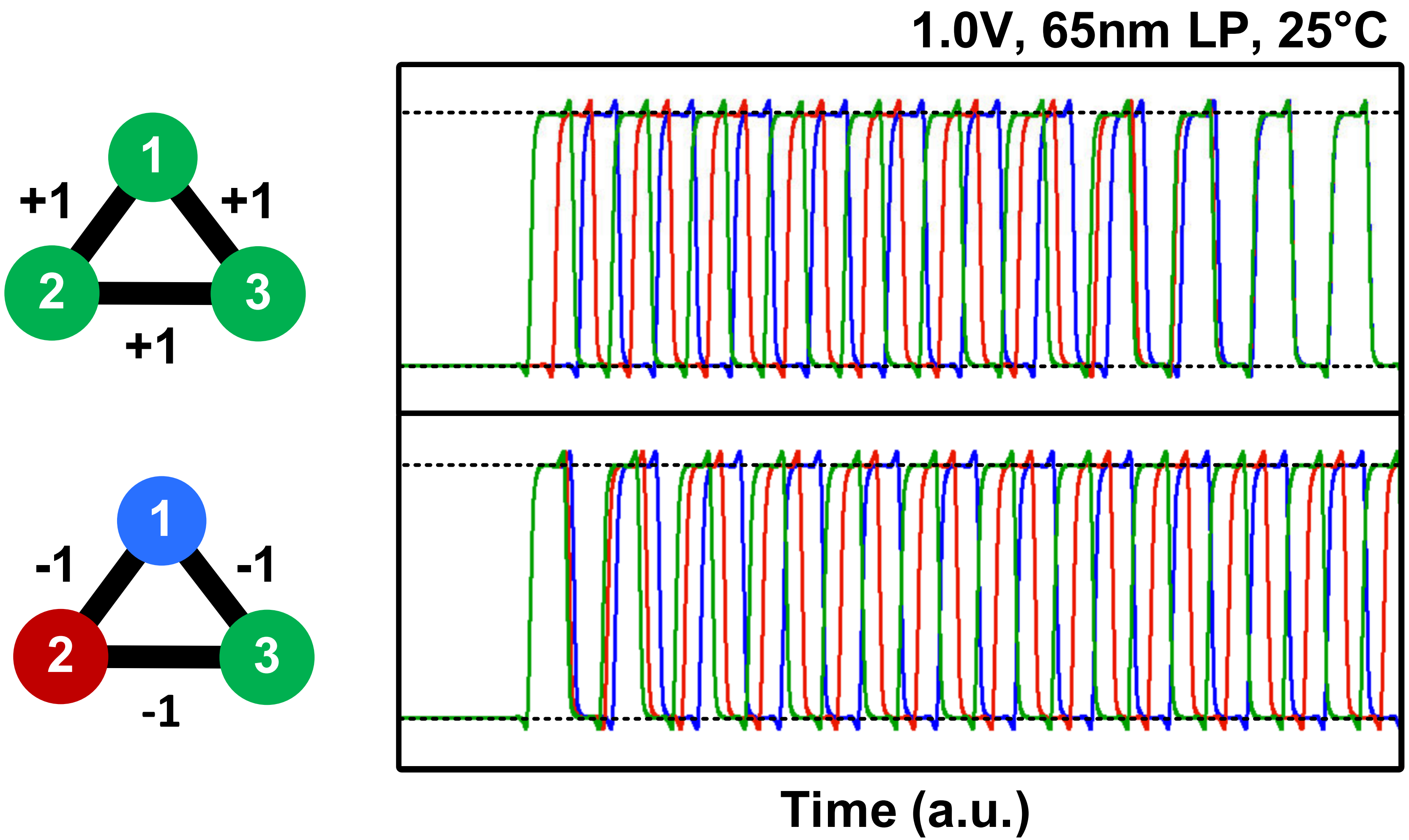
if $J_{ij} > 0$, then $\{s_i, s_j\}$={+1,+1} or {-1,-1}

: Same phase

if $J_{ij} < 0$, then $\{s_i, s_j\}$={+1,-1} or {-1,+1}

: Opposite phase

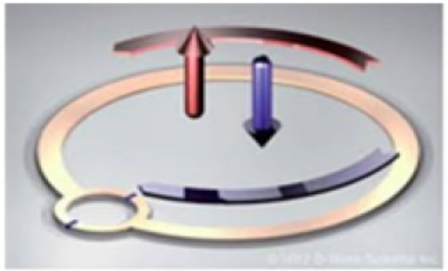
Using Coupled Oscillators to Find the Ground State



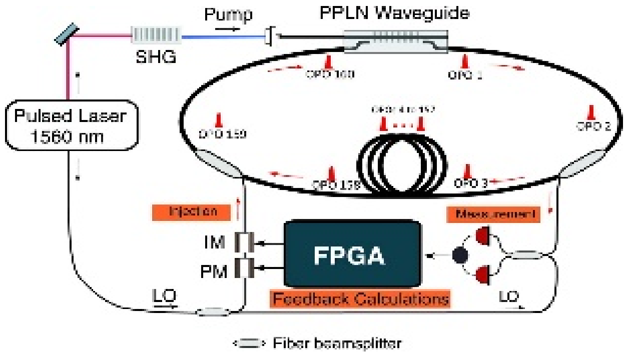
Final phases:
0°,0°,0°

Final phases:
-120°,0°,120°

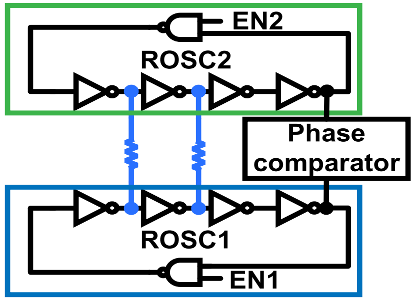
Other Oscillator Devices



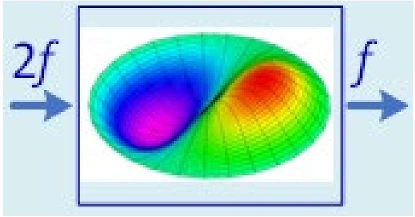
Superconducting qubits



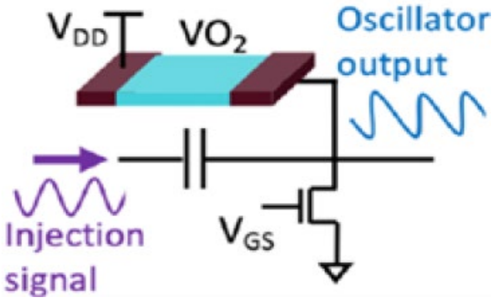
Optical fiber



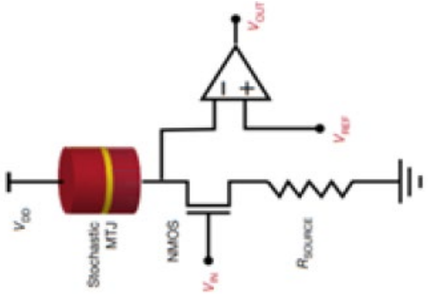
CMOS



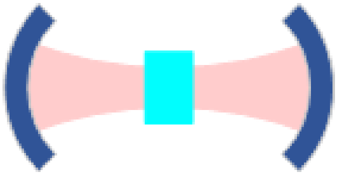
MEMS/NEMS



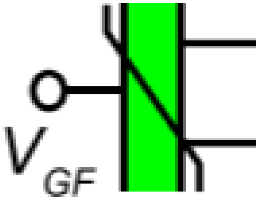
Phase transition material



Magnetic tunnel junctions



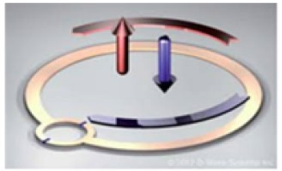
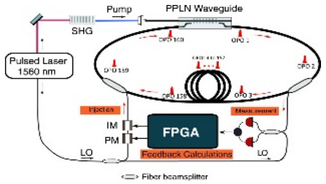
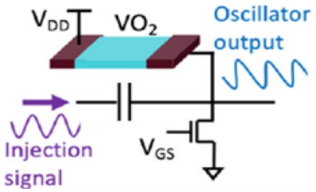
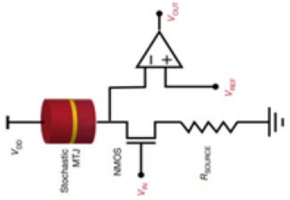
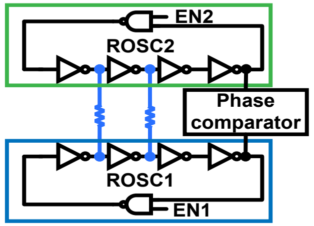
Cavity parametric oscillator



Ferroelectric

Sources: Google image, IEDM 20, EDL2017, Science 2016

Comparison of Coupled Oscillator Technologies

	Qubit	Optical	Phase transition	Spintronic	CMOS
Conceptual figure					
# of oscillators	~2000 (single chip)	100-2000 (lab setup)	4 (probe station)	8 (board level)	2000+ (1.3mm^2 chip in 65nm)
Advantages	Under debate	Room temperature	Room temperature	Room temperature	Room temperature, leverages CMOS, cloud/edge computing
Disadvantages	Cryogenic cool, 25kW power, premature tech. cloud only	1km optical fiber, FPGA chip, complex setup	Premature device, no real area advantage over CMOS	Premature device, no real area advantage over CMOS	Will it outperform GPUs and software solvers?
Integrated system in 10 yrs?	No	No	No	No	Yes
Target applications	NP-hard and NP-complete combinatorial optimization problems (e.g. supply chain, AI/ML, transportation, smart grid, communication, IC design, bioinformatics, computer vision, and robotics)				

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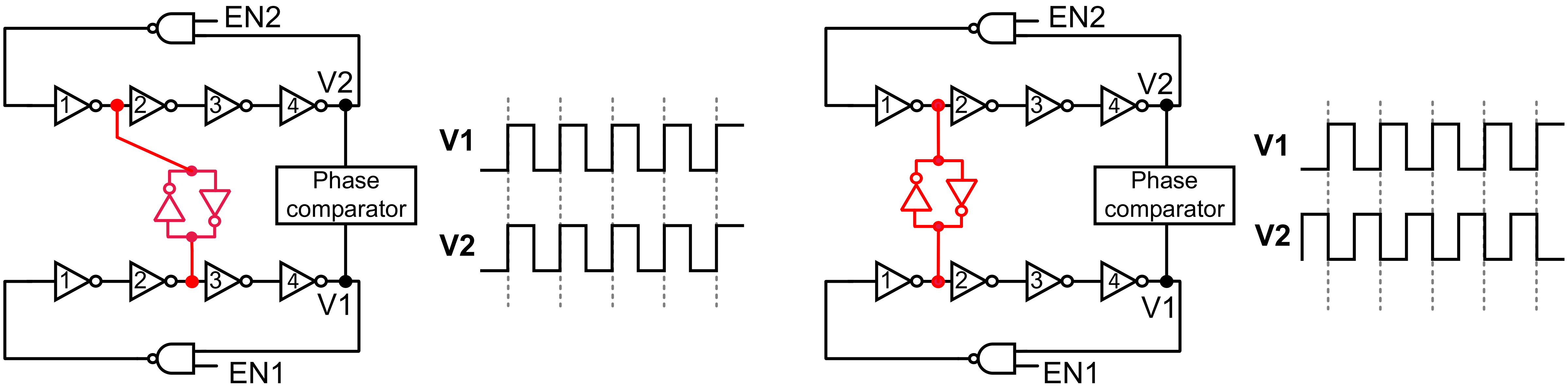
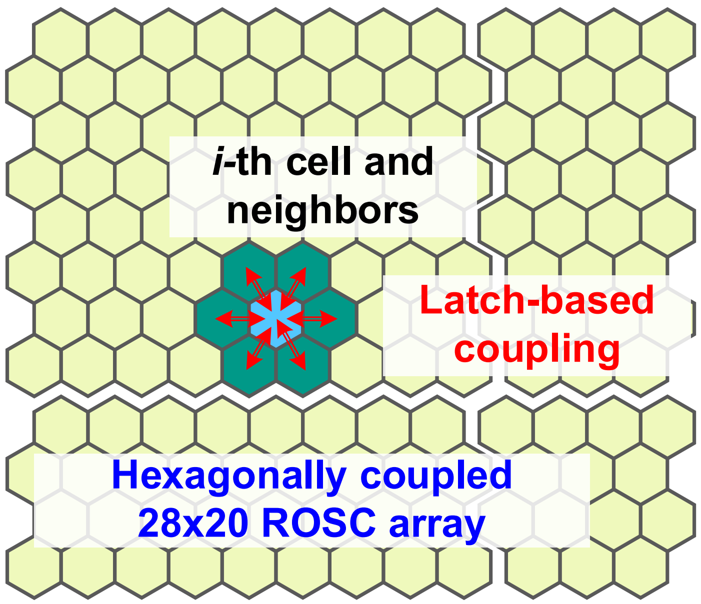
ROSC Coupled Using Digital Latches



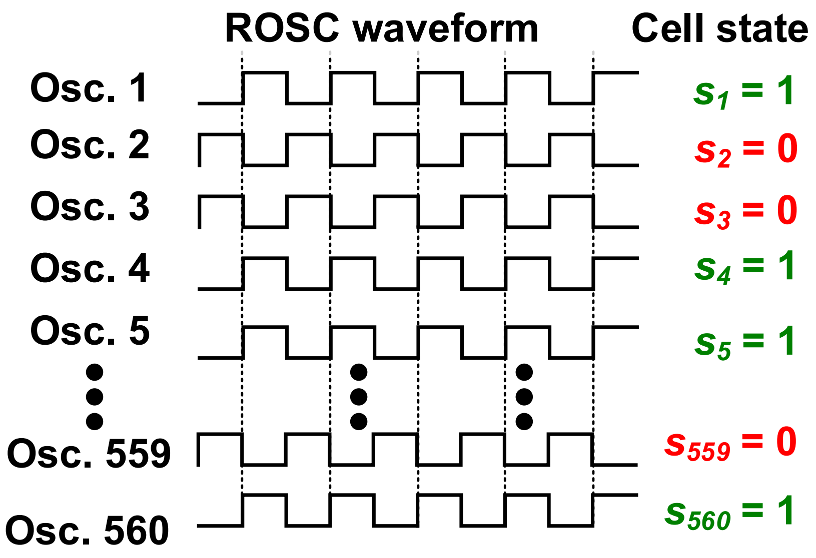
a) Positive coupling **b) Negative coupling**

- Any coupling medium that enables energy transfer may couple ROSCs
- ROSC and digital latches are designed with global and local enable signals

Choice of Architecture



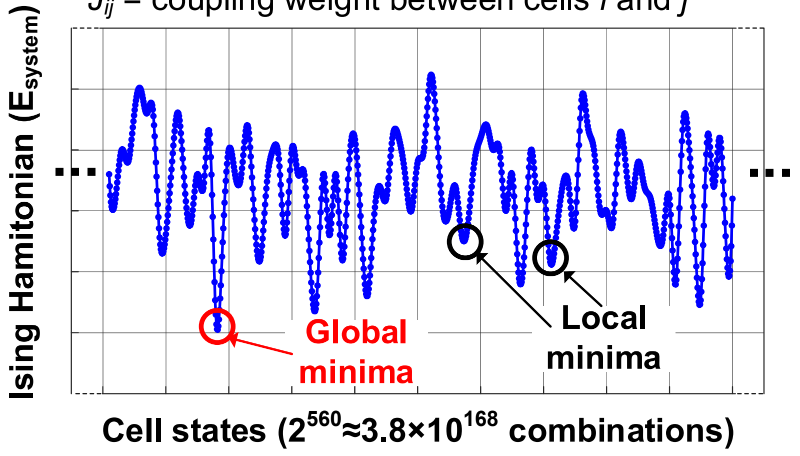
Proposed architecture



ROSC waveform and unit cell states

$$E_{system} = -\sum_{i=1}^{N}\sum_{j=1}^{n_i} J_{ij} \cdot s_i \cdot s_j$$

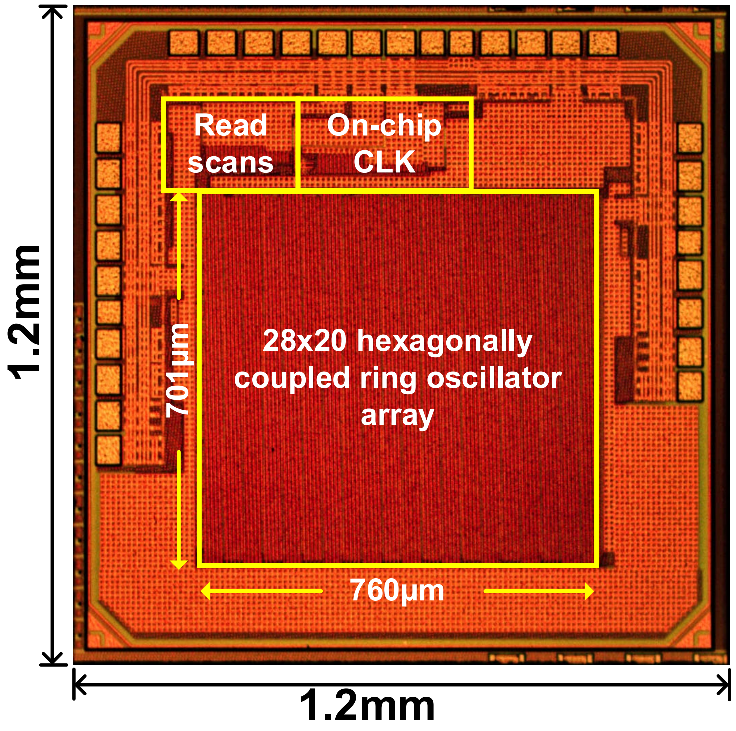
n_i = number of coupled neighbors of i-th cell,
J_{ij} = coupling weight between cells i and j



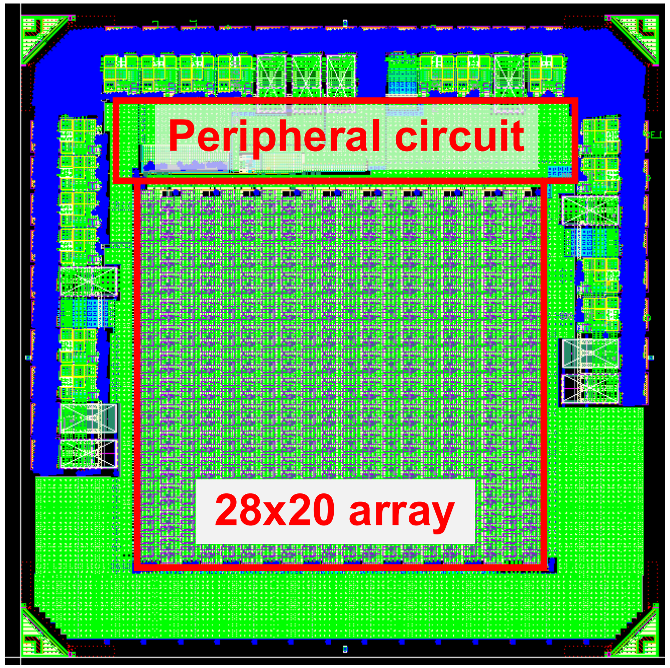
System energy of the Ising computer

- Hexagonal unit cell maximizes the number of neighbors in 2D plane
- Latch based coupling between cells is digitally controlled

Die Photo and Chip Summary



Die photo



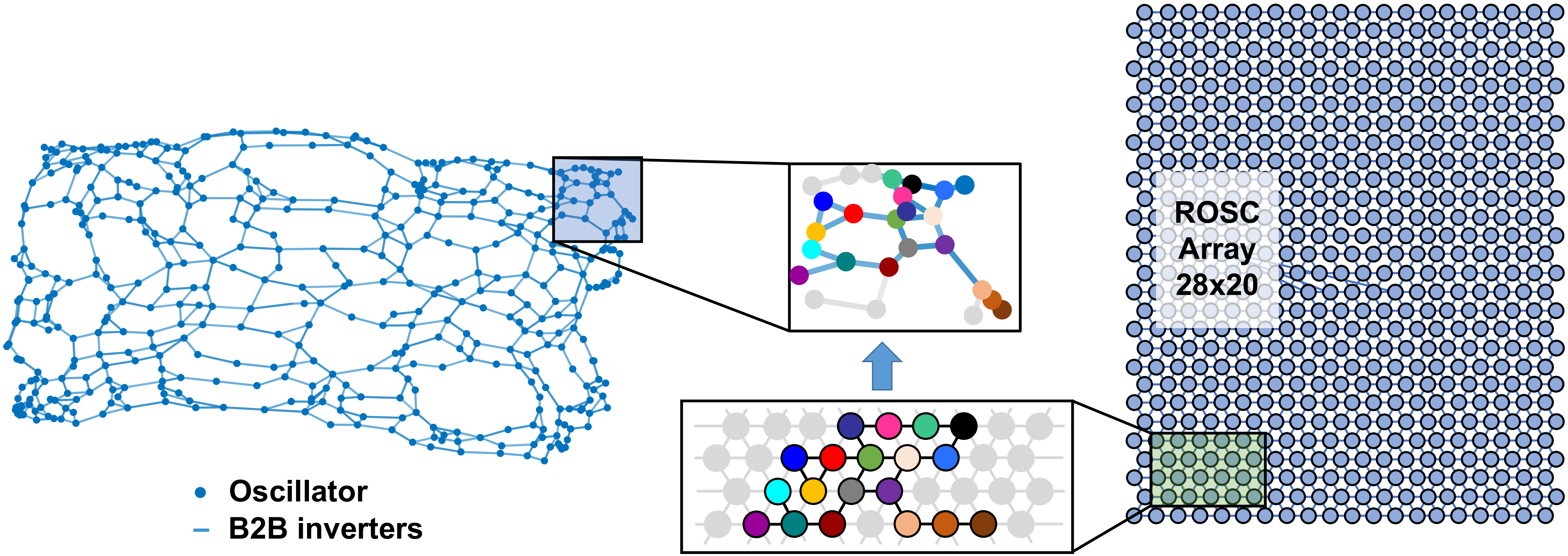
Full chip layout

Application	**Combinatorial optimization problems**
Process	**65nm CMOS**
Architecture	**ROSC, latch based coupling, self-annealing**
Voltage	**1.0V**
Area	**Chip: 1.44mm^2**
	Core: 0.53mm^2
	Unit cell: 0.00095mm^2
Peak power	**23mW**
Power per cell	**41μW**

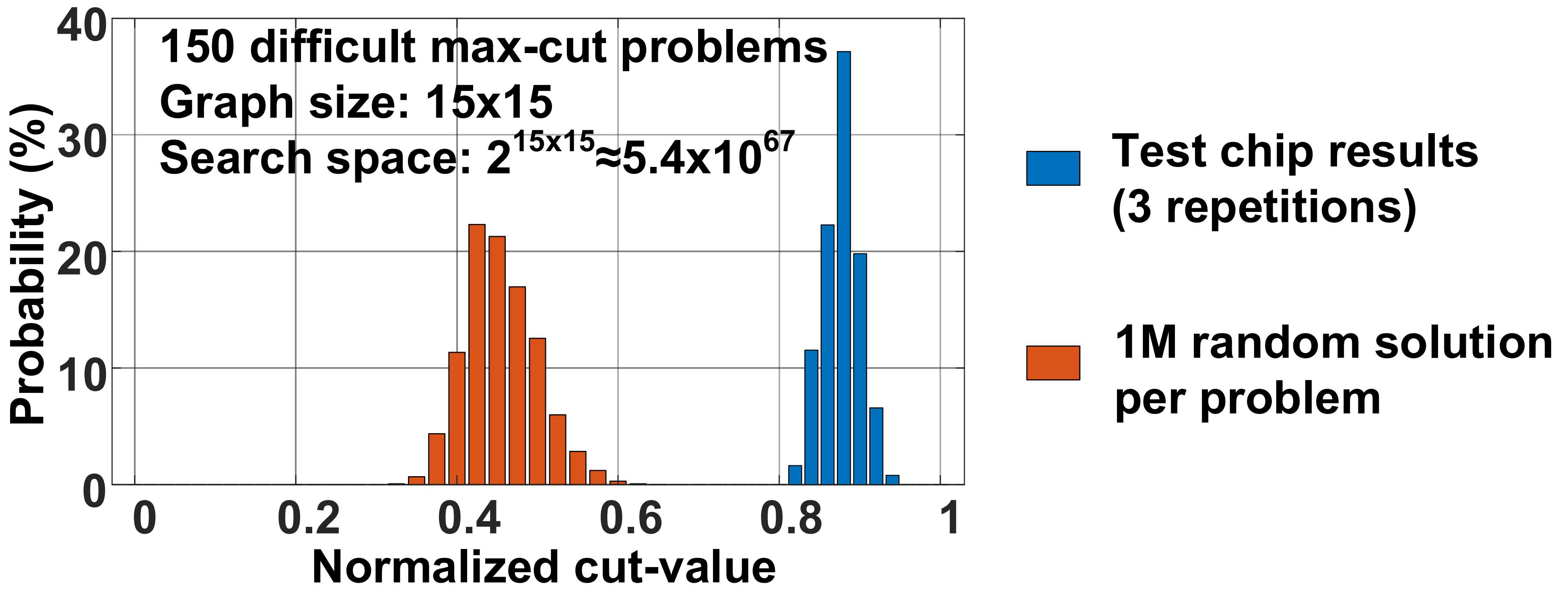
Chip summary

- 28x20=560 coupled oscillators (only limiting factor is chip area)
- Oscillator area < 5% of the full chip area

Embedding Ising Problem to Hardware

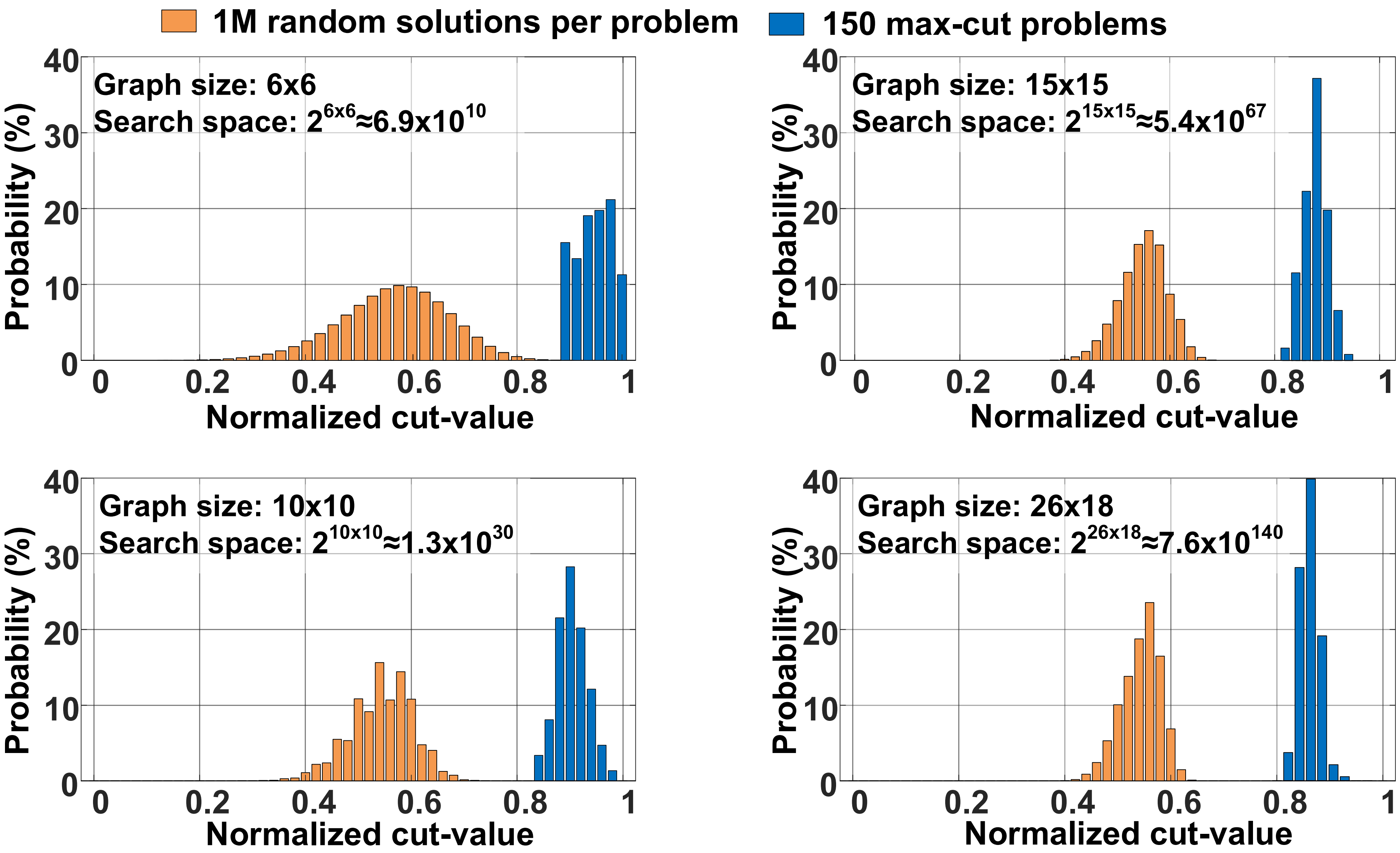


Max-cut Results for 15x15 Graphs

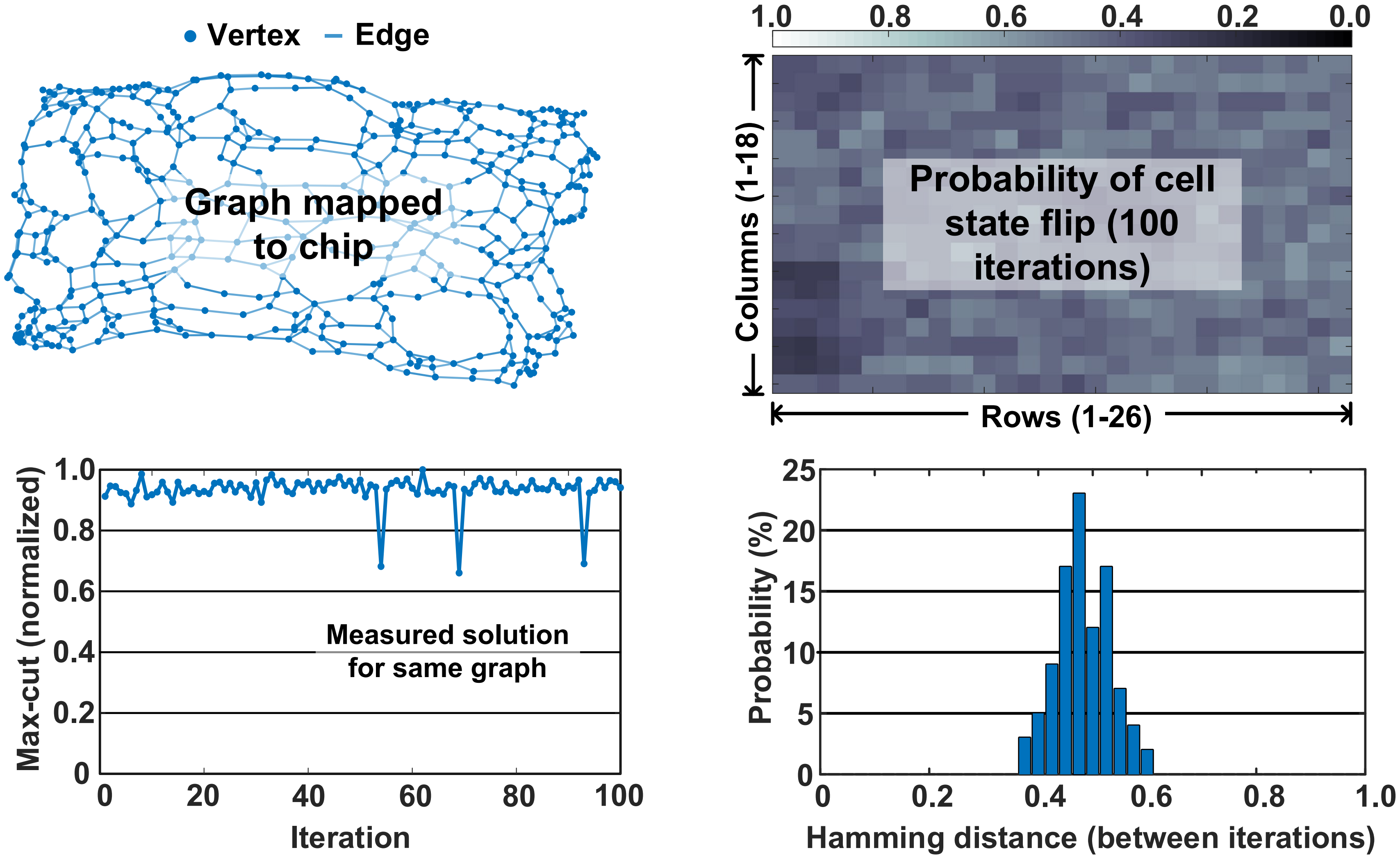


- 150 difficult COPs are mapped and max-cut results are measured for each graph sizes
- Measured results are compared with 1 million randomly sampled solutions from the solution-space for each specific graph.

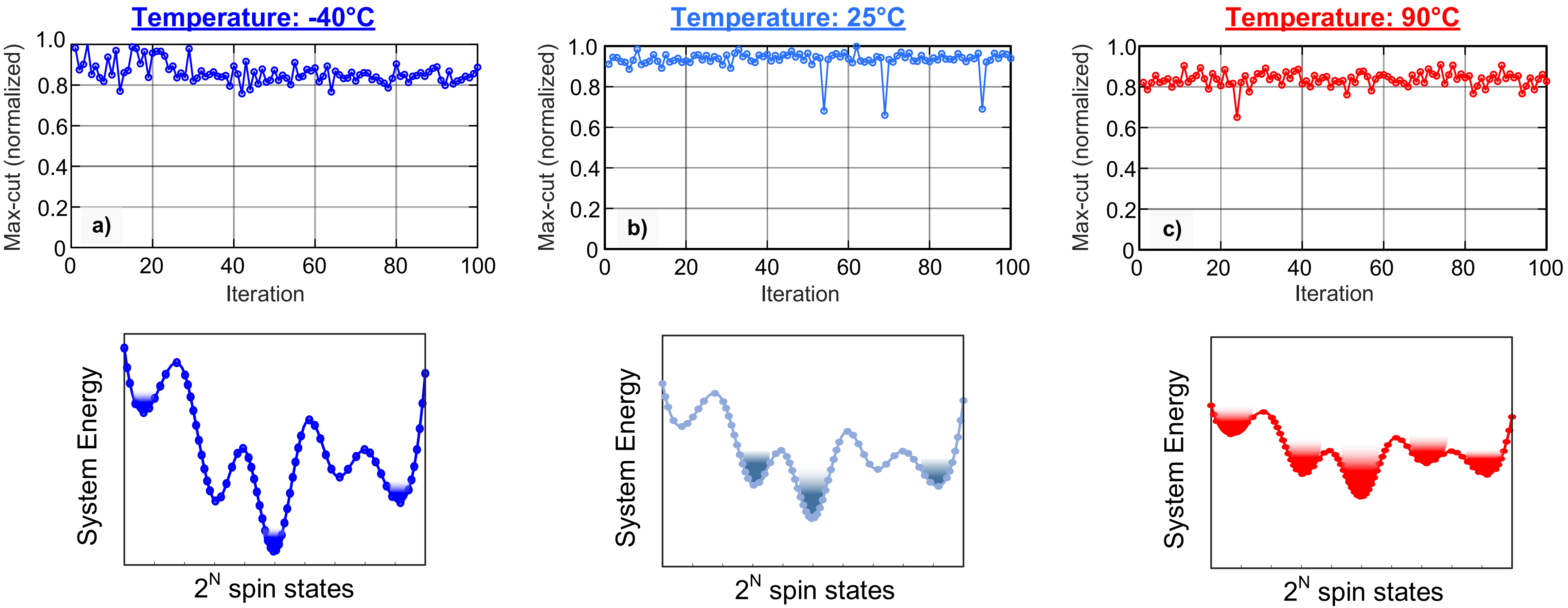
Max-Cut Results for Different Graph Sizes



Repeated Experiment for Same Graph



Temperature versus Solution Quality



Low temperature:
lower noise, stronger coupling

High temperature:
higher noise, weaker coupling

Takeaways

- NP-hard problems could be the key driver for future computing growth
- A true coupling based integrated CMOS Ising chip was demonstrated in 65nm
  - No emerging devices needed (old saying: anything that can be done in CMOS, will be done in CMOS)
  - Probabilistic exploration of various local minima
  - Mapped and solved 1,000 COPs in the chip with an accuracy of 82%-100%
- For oscillator based computing to be a viable approach however, there has to be a clear and significant power-performance-area advantage over
  - Mathematical optimizers (available today)
  - GPU, FPGA, Custom ASICs, digital annealers (available today)
  - Quantum computers