



IEEE Custom Integrated Circuits Conference

# Coupled Oscillator based Computing: Using Nature to Solve Difficult Problems

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*University of Minnesota*

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SOLID-STATE  
CIRCUITS SOCIETY™



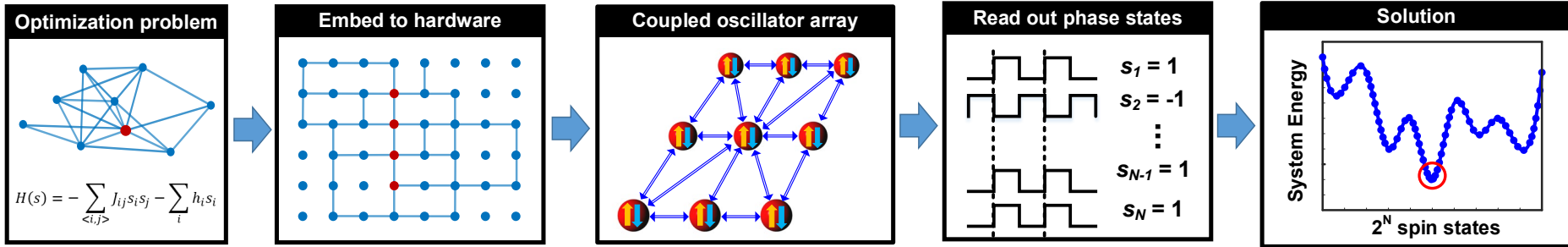
# Outline

- Introduction to Ising Computers
- Current State of the Art
- Case Study: 560 Coupled Oscillator Test Chip
- Commercialization Effort and Outlook
- Summary

## Context and Disclaimer

- This presentation focuses on
  - a new computing paradigm
  - in early stages of research
  - that is highly exploratory and somewhat finicky
  - with very little experimental data available in public domain
- Sit back and relax (with an open mind)

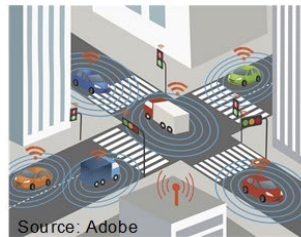
# Ising Spin Glass Model



- A promising approach for efficiently solving NP-hard or NP-complete problems (e.g. combinatorial optimization problems, Boltzman machines, associative memories, Karp's 21 NP-complete problems)

$$H(s) = - \sum_{\langle i,j \rangle} J_{ij} s_i s_j - \sum_i h_i s_i$$
 : Ising Hamiltonian (Cost Function)  
 $s_i, s_j$  : Spin state  $\{+1 \text{ or } -1\}$      $J_{ij}$  : Coupling strength     $h_i$  : local field strength

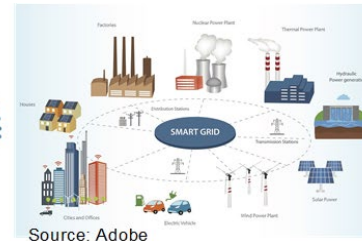
Autonomous vehicles



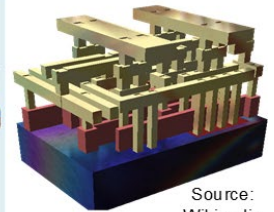
Communication networks



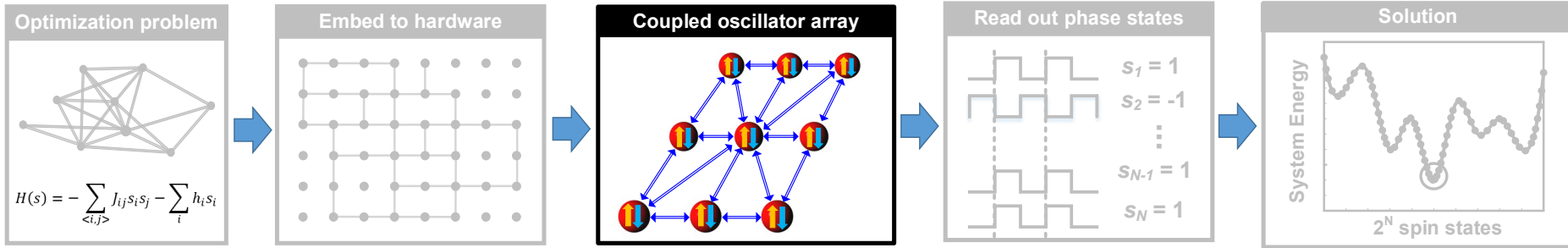
Smart grid



VLSI routing



# Using Nature to Find the Ground State



Random states (time=0)



Same states (time = 1 min)

# Example Problem #1: Factorizing 15

$$p = (x_1 \ 1)_2, q = (x_2 \ x_3 \ 1)_2$$

$$H = (15 - pq)^2$$

$$H = 128x_1x_2x_3 - 56x_1x_2 - 48x_1x_3 + 16x_2x_3 - 52x_1 - 52x_2 - 96x_3 + 196$$

$$H_{mod} = 200x_1x_2 - 48x_1x_3 - 512x_1x_4 + 16x_2x_3 - 512x_2x_4 + \\ 128x_3x_4 - 52x_1 - 52x_2 - 96x_3 + 768x_4 + 196$$

S. Jiang, et al., "Quantum Annealing for Prime Factorization", Scientific Reports 2018

# Example Problem #2: Graph Coloring

For graph  $G(V, E)$  of the map problem—no two vertices,  $V$ , connected by an edge,  $E$ , should select the same color from set  $C$ —construct a cost function with binary variables,  $x_{v,c} = 1$  when  $v \in V$  selects color  $c \in C$ , by implementing two constraints:

$$\left(\sum_c x_{v,c} - 1\right)^2,$$

which has minimum energy (zero) when vertices select one color only, and

$$\sum_c \sum_{v_a, v_b \in E} x_{v_a, c} x_{v_b, c},$$

which adds a penalty if the vertices of an edge select the same color.

These constraints give a QUBO,

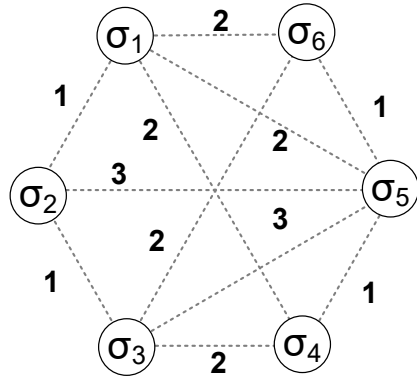
$$E(x_v, x_{v_a, v_b}) = \sum_v \left(\sum_c x_{v,c} - 1\right)^2 + \sum_c \sum_{v_a, v_b \in E} x_{v_a, c} x_{v_b, c}.$$

e.g.  $x_{Minn, Red} = 0, x_{Minn, Blue} = 0,$   
 $x_{Minn, Sand} = 1, x_{Minn, Green} = 0$   
 $x_{Wisc, Red} = 0, x_{Wisc, Blue} = 1,$   
 $x_{Wisc, Sand} = 0, x_{Wisc, Green} = 0$

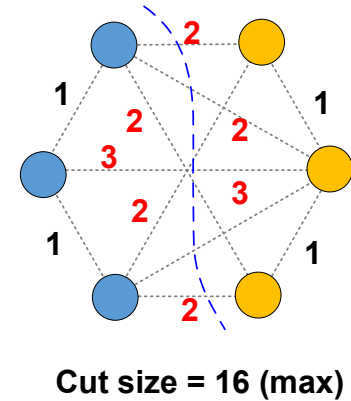
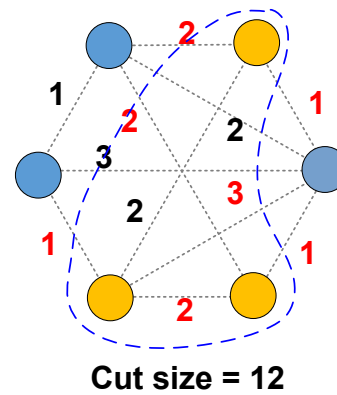
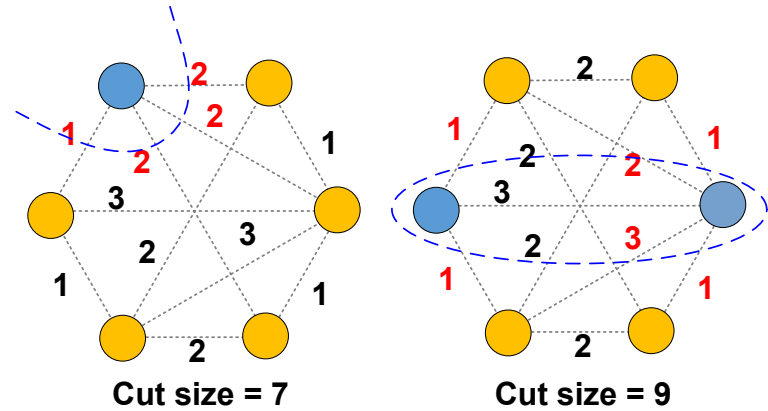


D-Wave Problem-Solving Handbook, 8/13/20

# Example Problem #3: Finding Max-cut

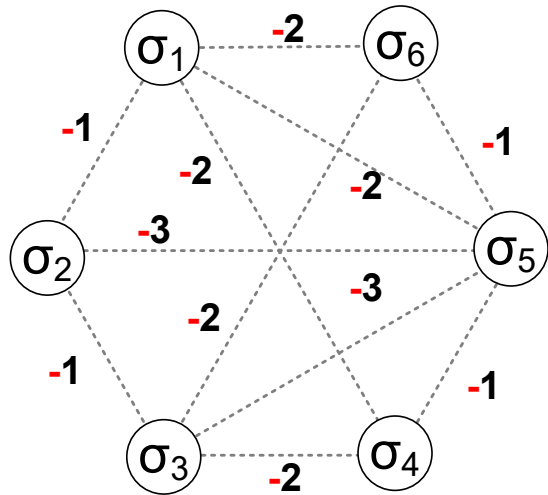


- The problem of finding a maximum cut in a graph is known as the Max-Cut Problem
- Finding max-cut of a graph is an *NP-hard* problem





# Example Problem #3: Finding Max-cut



$$\begin{aligned}
 H(\sigma) &= - \sum_{i,j} (-w_{ij}) \sigma_i \sigma_j \\
 &= \sum_{diff\ group} (-w_{ij}) + \sum_{same\ group} w_{ij} \\
 &= \sum_{diff\ group} (-w_{ij}) + \left[ \sum_{i,j} w_{ij} - \sum_{diff\ group} w_{ij} \right] \\
 &= \sum_{all} w_{ij} - 2 \times \underbrace{\sum_{diff\ group} w_{ij}}_{\text{Cut size}}
 \end{aligned}$$

$H$  = Hamiltonian of the system  
 $\sigma_i$  = Spin status of magnet  $i$  {+1 or -1}  
 $w_{ij}$  = weight between magnets  $i$  and  $j$

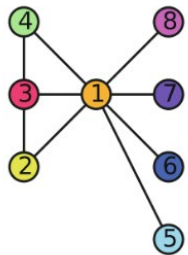
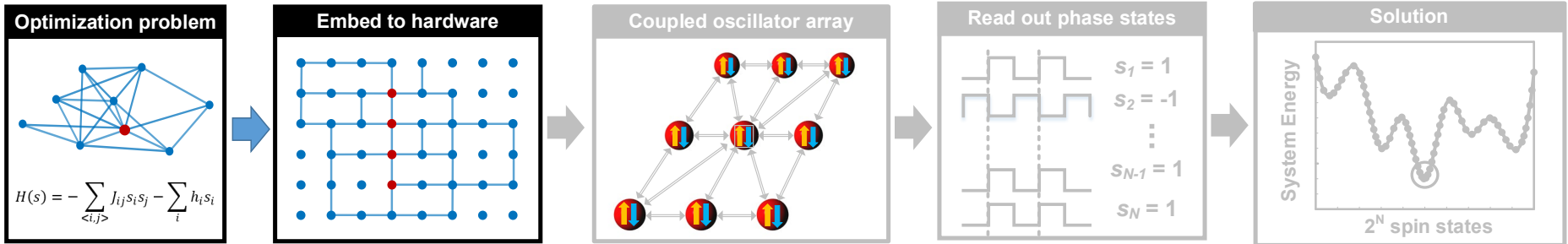
- Ising Hamiltonian = [sum of all weights] - 2×[cut size]

# Other NP Problems Mappable to the Ising Model

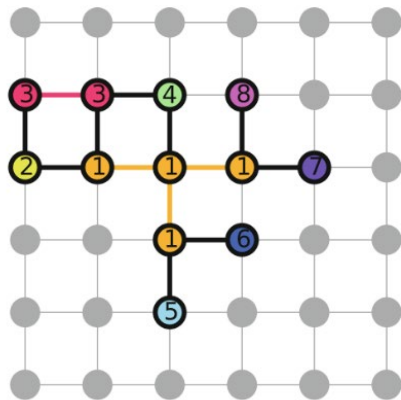
- Partitioning problems (e.g. max cut)
- Binary integer linear programming
- Covering and packing problems
- Problems with inequalities
- Coloring problems (e.g. graph coloring)
- Hamiltonian cycles (e.g. traveling salesman)
- Tree problems
- Graph isomorphisms
- ...

A. Lucas, "Ising formulations of many NP problems", *Frontiers in Physics*, Feb. 2014

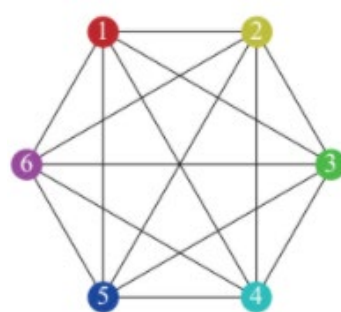
# Example of Graph Embedding



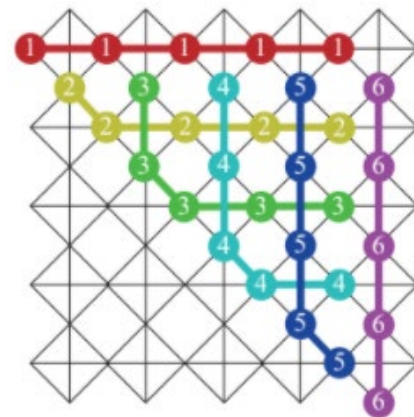
G



$G_{emb}$

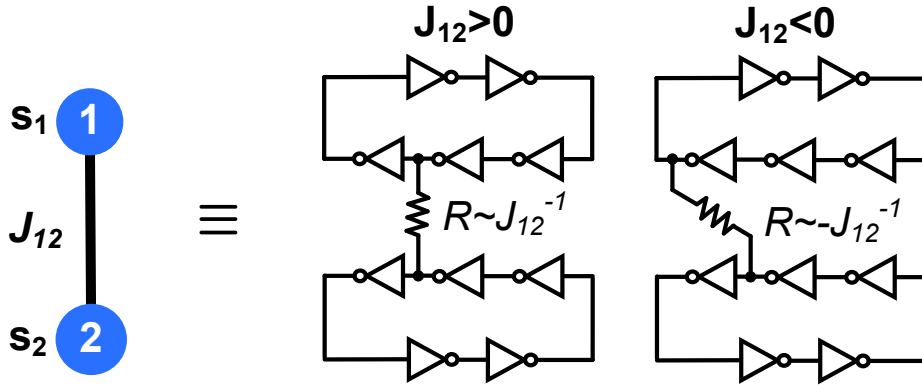
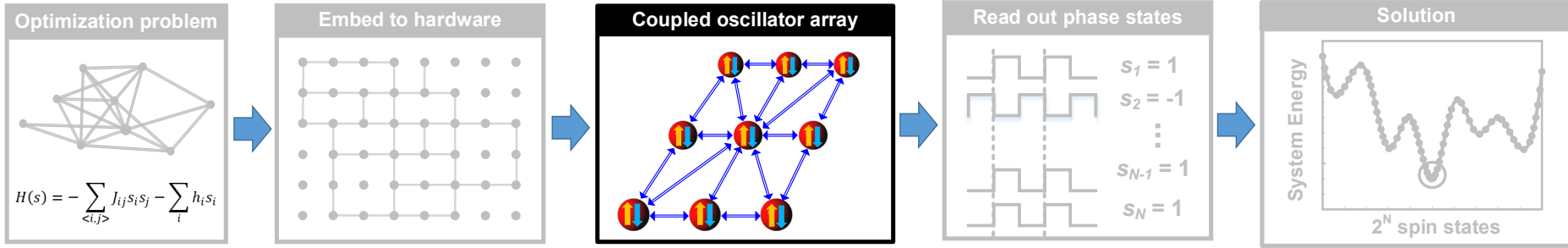


6 spin all-to-all



King's architecture

# Using Coupled Oscillators to Find the Ground State

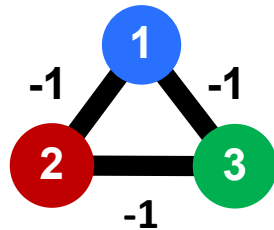
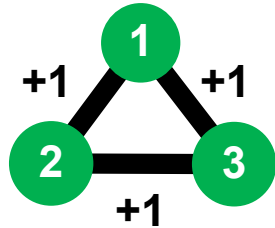


$$H(s) = -J_{ij} s_i s_j$$

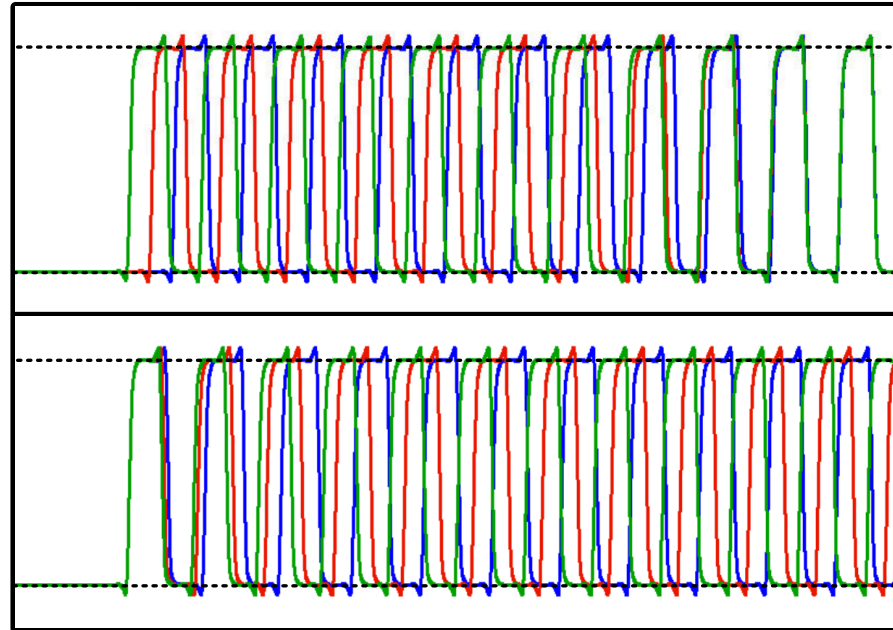
if  $J_{ij} > 0$ , then  $\{s_i, s_j\} = \{+1, +1\}$  or  $\{-1, -1\}$ : Same phase

if  $J_{ij} < 0$ , then  $\{s_i, s_j\} = \{+1, -1\}$  or  $\{-1, +1\}$ : Opposite phase

# Using Coupled Oscillators to Find the Ground State



1.0V, 65nm LP, 25°C

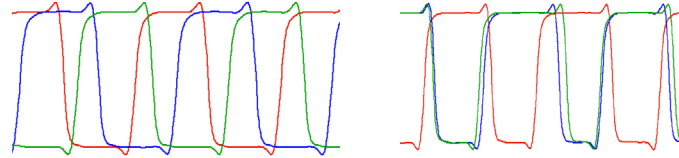


Final phases:  
 $0^\circ, 0^\circ, 0^\circ$

Final phases:  
 $-120^\circ, 0^\circ, 120^\circ$

Time (a.u.)

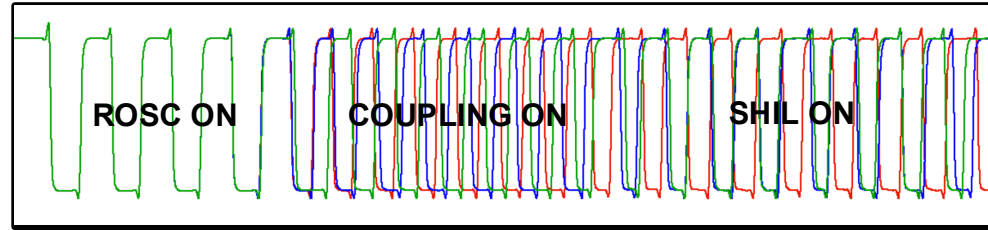
# Super Harmonic Injection Locking (SHIL) Signal



Phases:  
 $0^\circ, 0^\circ, 0^\circ$

Phases:  
 $-120^\circ, 0^\circ, 120^\circ$

Phases:  
 $0^\circ, 180^\circ, 180^\circ$

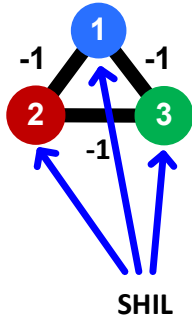


ROSC ON

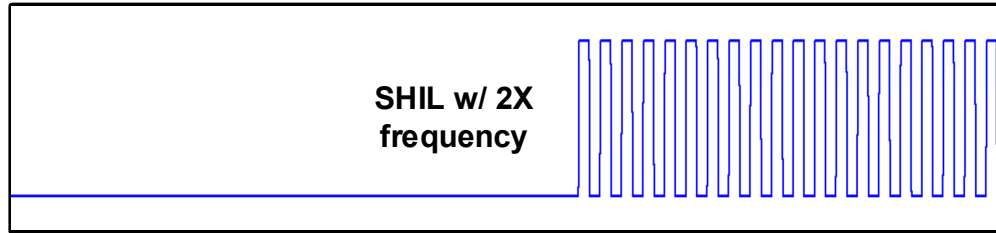
COUPLING ON

SHIL ON

Time (a.u.)



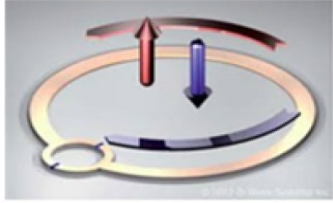
SHIL



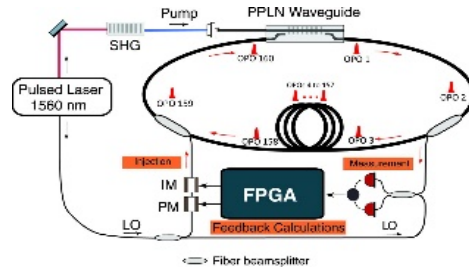
SHIL w/ 2X  
frequency

Time (a.u.)

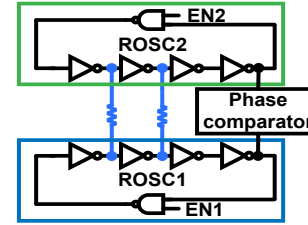
# Other Oscillator Devices



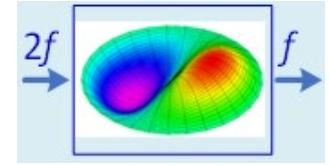
Superconducting qubits



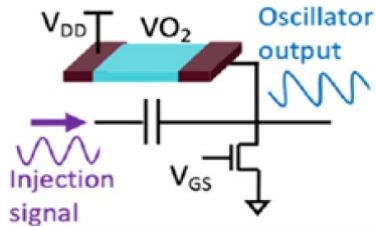
Optical fiber



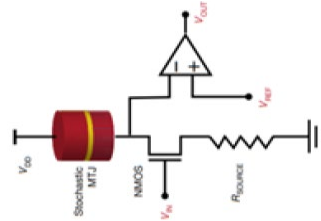
CMOS



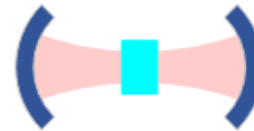
MEMS/NEMS



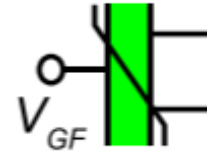
Phase transition material



Magnetic tunnel junctions



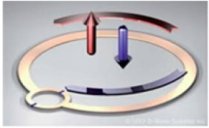
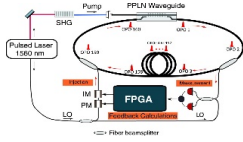
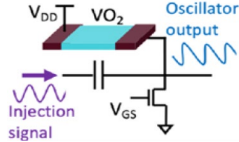
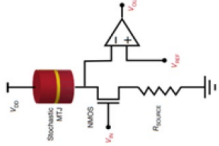
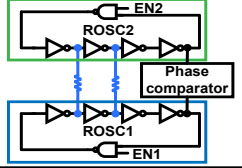
Cavity parametric oscillator



Ferroelectric

Sources: Google image, IEDM 20, EDL2017, Science 2016

# Comparison of Coupled Oscillator Technologies

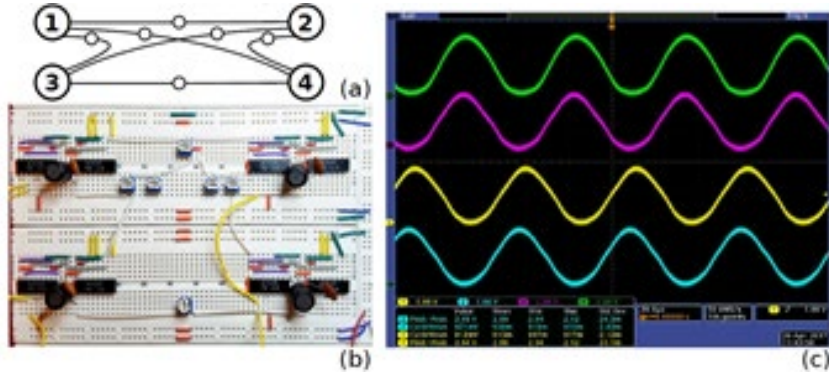
	Qubit	Optical	Phase transition	Spintronic	CMOS
Conceptual figure					
# of oscillators	~2000 (single chip)	100-2000 (lab setup)	4 (probe station)	8 (board level)	2000+ (1.3mm <sup>2</sup> chip in 65nm)
Advantages	Under debate	Room temperature	Room temperature	Room temperature	Room temperature, leverages CMOS, cloud/edge computing
Disadvantages	Cryogenic cool, 25kW power, premature tech. cloud only	1km optical fiber, FPGA chip, complex setup	Premature device, no real area advantage over CMOS	Premature device, no real area advantage over CMOS	Will it outperform GPUs and software solvers?
Integrated system in 10 yrs?	No	No	No	No	Yes
Target applications	NP-hard and NP-complete combinatorial optimization problems (e.g. supply chain, AI/ML, transportation, smart grid, communication, IC design, bioinformatics, computer vision, and robotics)				



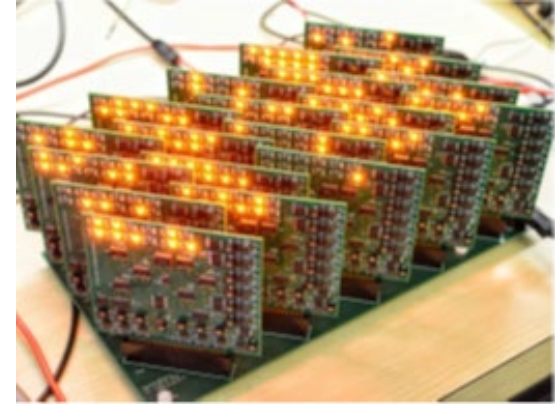
# Outline

- Introduction to Ising Computers
- **Current State of the Art**
- Case Study: 560 Coupled Oscillator Test Chip
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# Breadboard and PCB Implementation



T. Wang, *arXiv*, 2017 (UC Berkeley)



T. Wang, *arXiv*, 2019 (UC Berkeley)

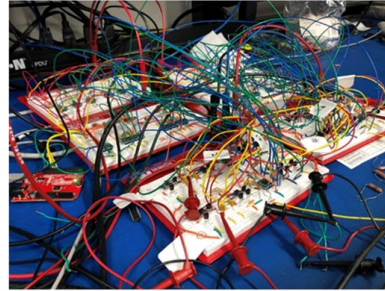
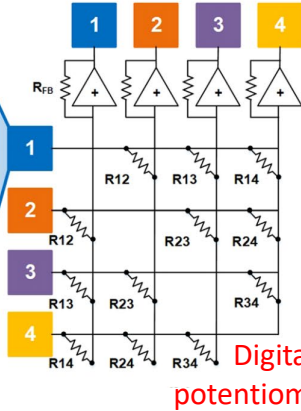
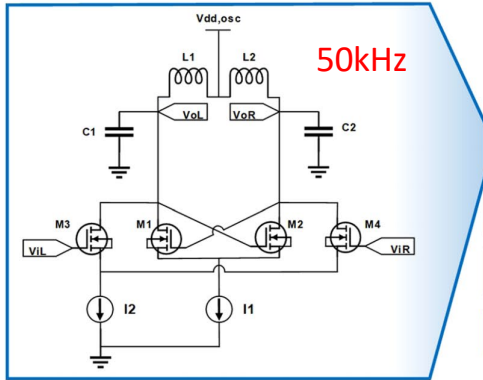
## Pros:

Can achieve coupling dynamics, good for proving the concept

## Cons:

Not practical for large number of programmable spins, not an integrated solution

# Breadboard and PCB Implementation



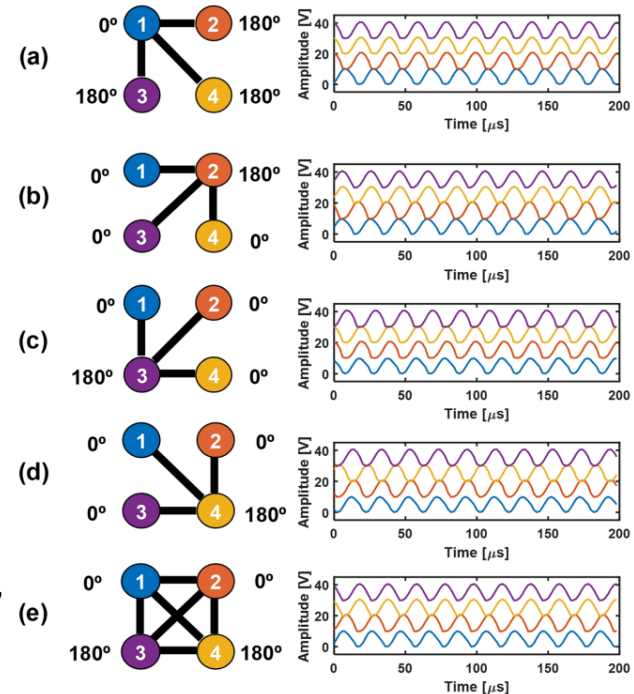
J. Chou, S. Bramhavar, et al., Scientific Reports, 2019 (MIT Lincoln Labs)

## Pros:

Can achieve coupling dynamics, good for proving the concept, **all-to-all coupling using crossbar architecture**

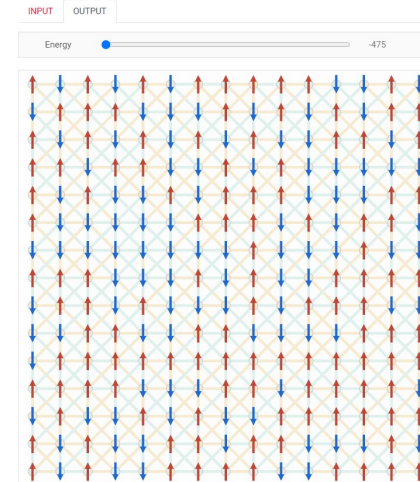
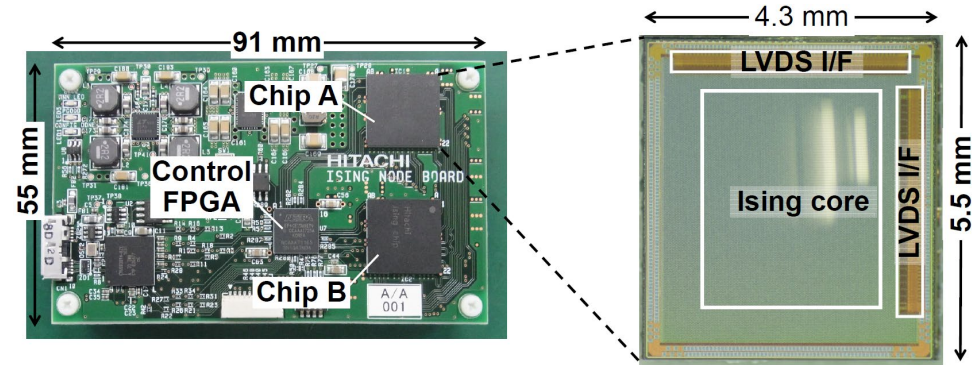
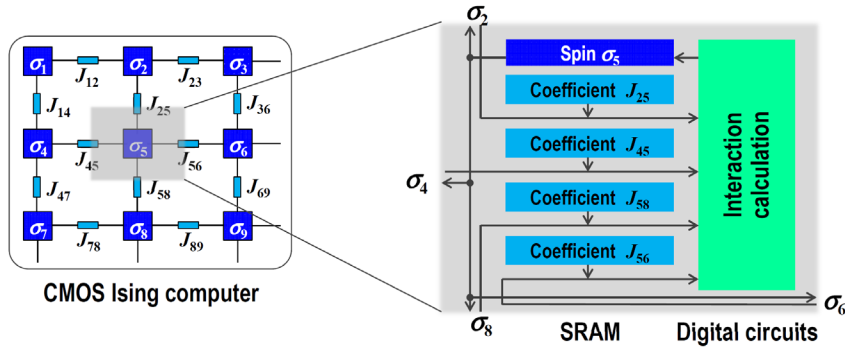
## Cons:

Not practical for large number of programmable spins, not an integrated solution



# ASIC Implementation

M. Yamaoka, *JSSC*, 2016 (Hitachi)  
T. Takemoto, *ISSCC*, 2019 (Hitachi)



Real time demo available at  
[www.annealing-cloud.com](http://www.annealing-cloud.com)

## Pros:

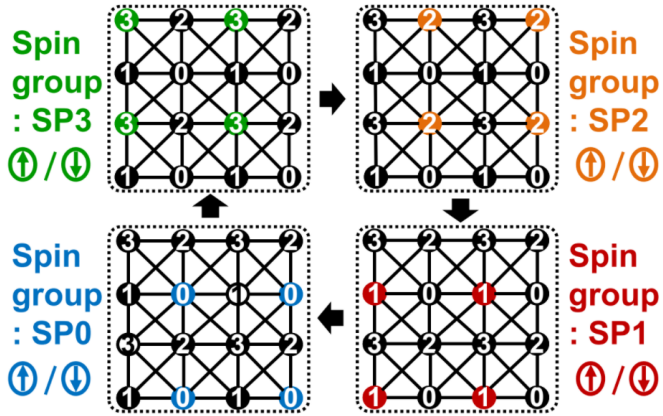
Can theoretically solve a wide variety of problems

## Cons:

1. No coupling dynamics : computation time and energy may be higher
2. Search for better solution is time dependent

# ASIC Implementation

T. Takemoto, *ISSCC*, 2019 (Hitachi)



## Pros:

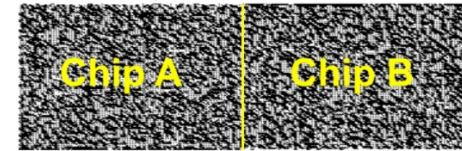
Can theoretically solve a wide variety of problems

## Cons:

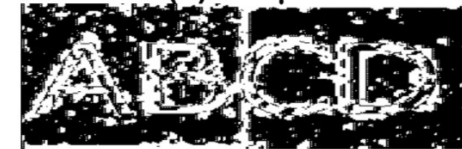
1. No coupling dynamics : computation time and energy may be higher
2. Search for better solution is time dependent

	D-Wave 2000Q	ISSCC2015 (previous)	This work
Method	Quantum annealing	Simulated annealing	
Accuracy	Better	Not so good	Good
Implementation	Superconductor	65-nm CMOS	40-nm CMOS
Number of connected chips	No	No	2 (multichip in principle)
Number of spins	2k	20k	2 × 30k
Bit width of coefficients	N/A	2 bits	3 bits
Annealing time*	N/A	10 ms***	22 μs
Energy efficiency*, **	N/A	2200 times***	1.75 × 10 <sup>5</sup> times

(i) 0 μs (initial state)



(ii) 7.7 μs



(iii) 21.8 μs (final state)



# ASIC Implementation

Max Scale: **8192 bit**  
 Max Precision: **64 bit**  
 1.845x10<sup>19</sup> Gradations

Digital Annealing Unit

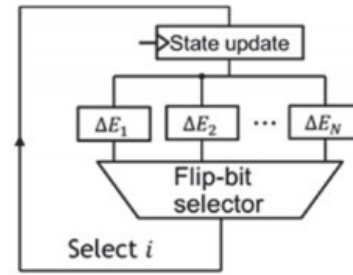
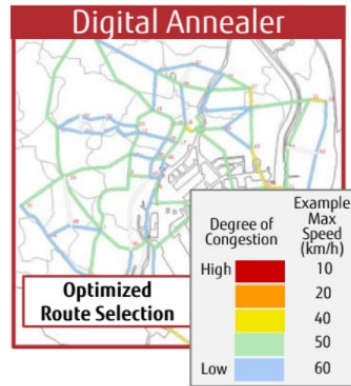
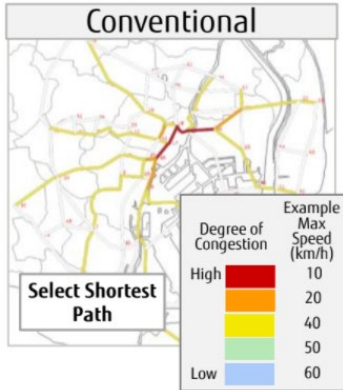


Fig. 1. A diagram of the parallel search technique

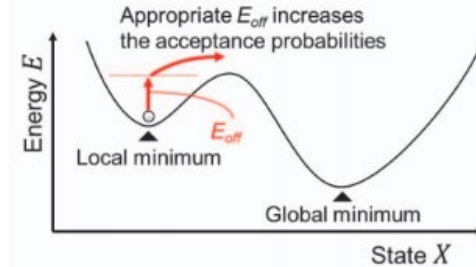
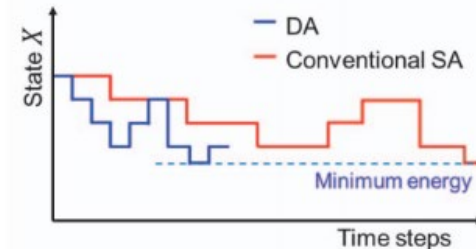


Fig. 2. A diagram of the escape technique



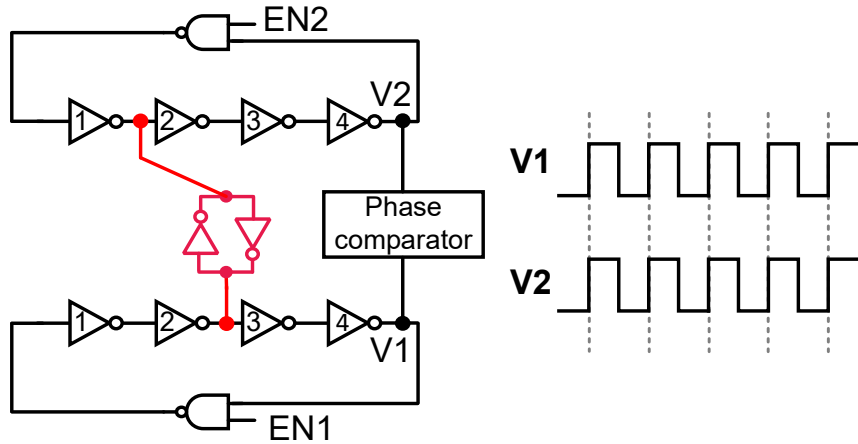
S. Matsubara, et al., ASP-DAC 2020 (Fujitsu)

Fig. 3. Conceptual diagram of speed-up achieved by DA

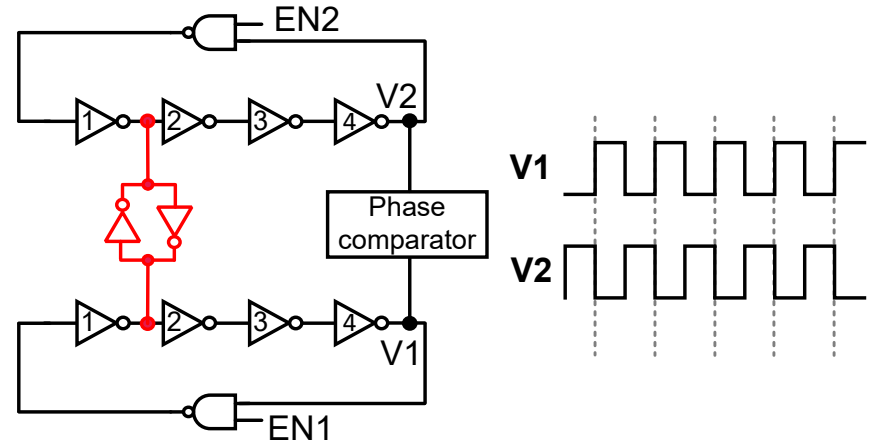
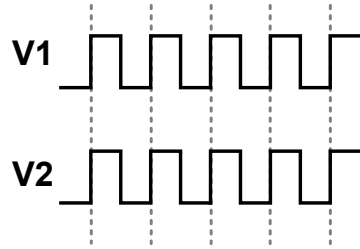
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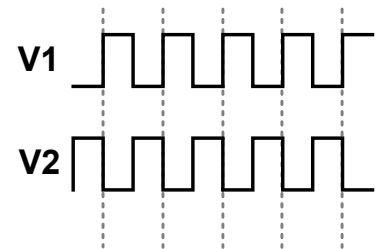
# ROSC Coupled Using Digital Latches



a) Positive coupling



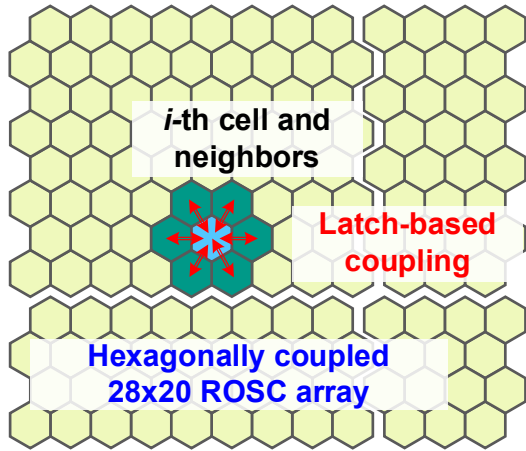
b) Negative coupling



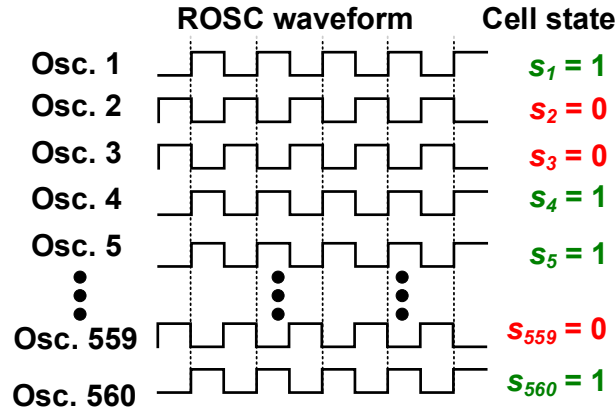
- Any coupling medium that enables energy transfer may couple ROSCs
- ROSC and digital latches are designed with global and local enable signals



# Choice of Architecture



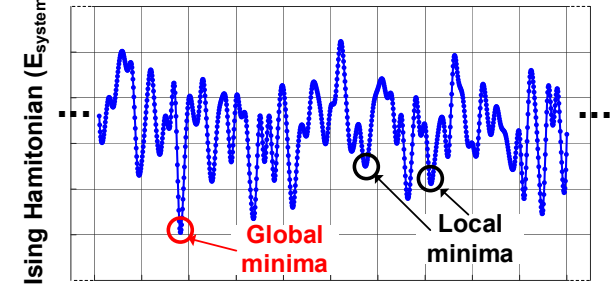
Proposed architecture



ROSC waveform and unit cell states

$$E_{system} = - \sum_{i=1}^N \sum_{j=1}^{n_i} J_{ij} \cdot s_i \cdot s_j$$

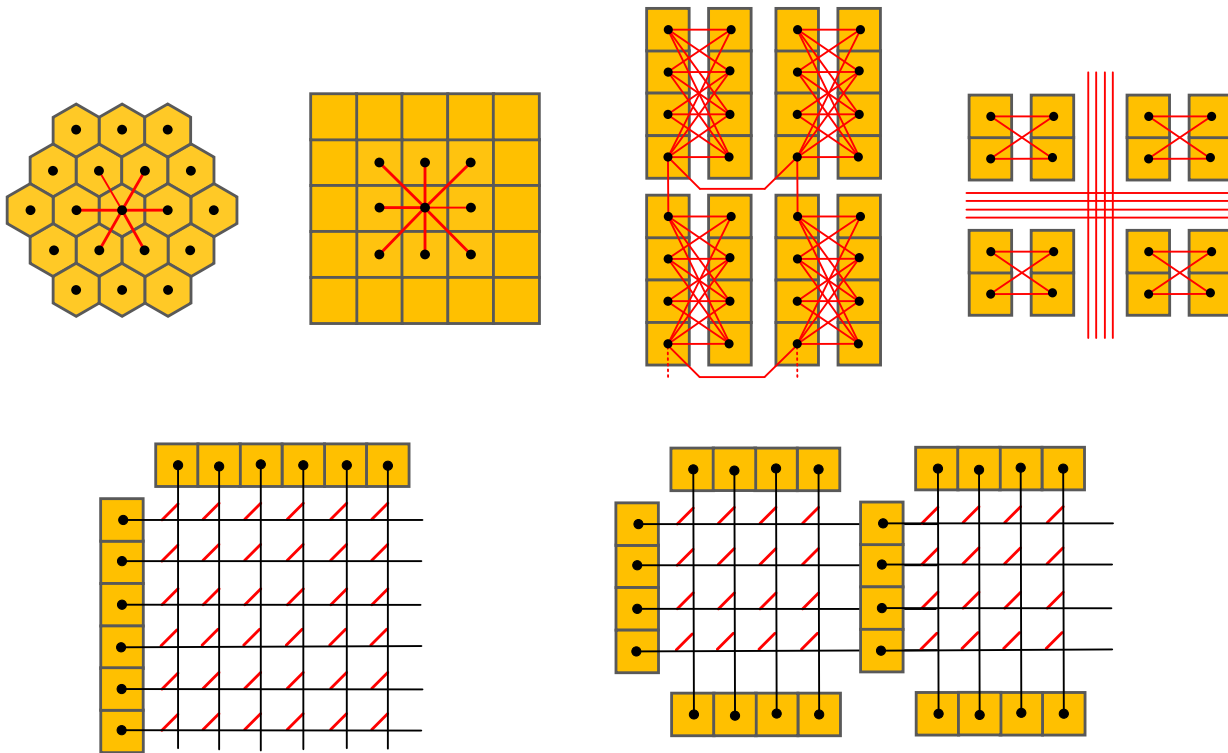
$n_i$  = number of coupled neighbors of  $i$ -th cell,  
 $J_{ij}$  = coupling weight between cells  $i$  and  $j$



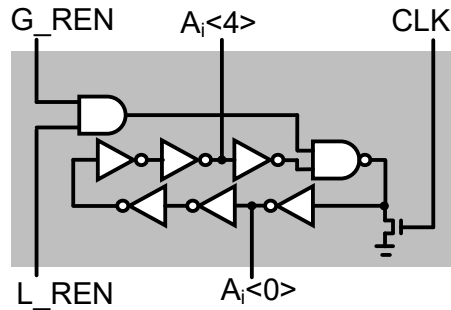
System energy of the Ising computer

- Hexagonal unit cell maximizes the number of neighbors in 2D plane
- Latch based coupling between cells is digitally controlled

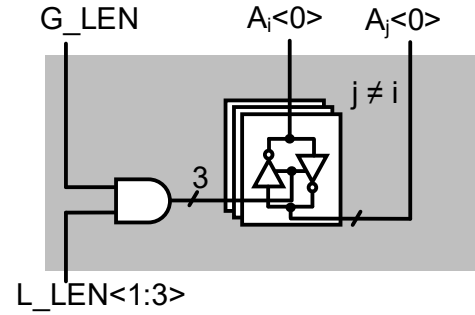
# Other Possible Architectures



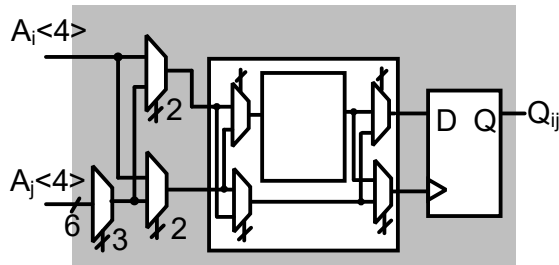
# Modular Unit Cell: Circuit Blocks



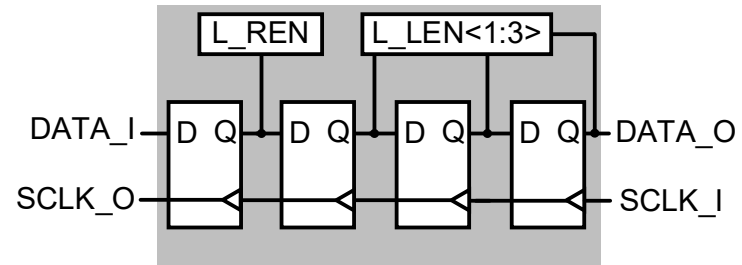
**a) ROsc block**



**b) Latch coupling block**



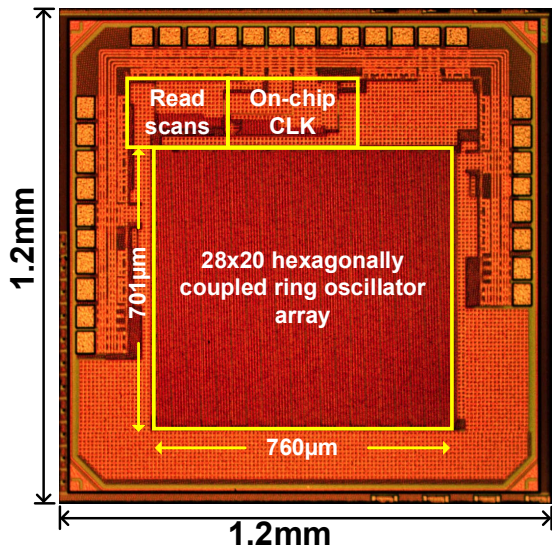
**c) Read block**



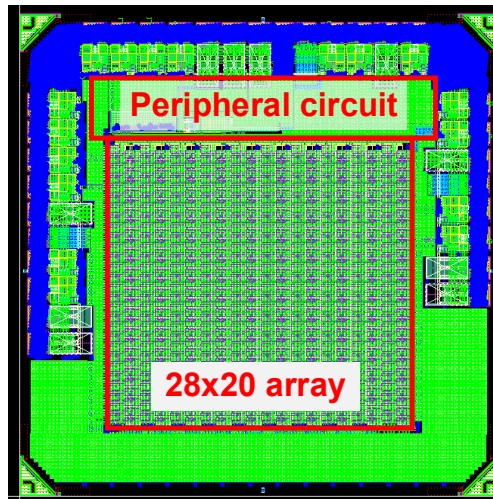
**d) Scan block**

- Read block samples one of the neighboring ROsc
- Scan block programs the graph: four program bits per cell, 2,240 bits for the chip

# Die Photo and Chip Summary



Die photo



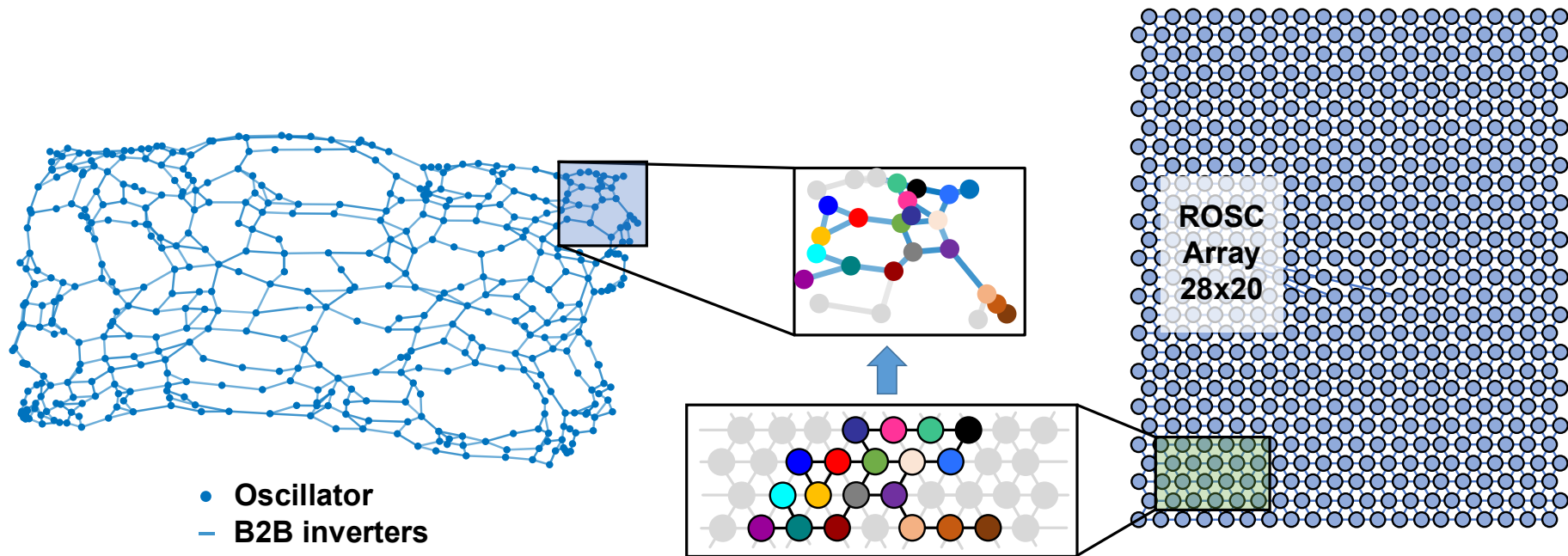
Full chip layout

Application	Combinatorial optimization problems
Process	65nm CMOS
Architecture	ROSC, latch based coupling, self-annealing
Voltage	1.0V
Area	Chip: 1.44mm <sup>2</sup>
	Core: 0.53mm <sup>2</sup>
	Unit cell: 0.00095mm <sup>2</sup>
Peak power	23mW
Power per cell	41μW

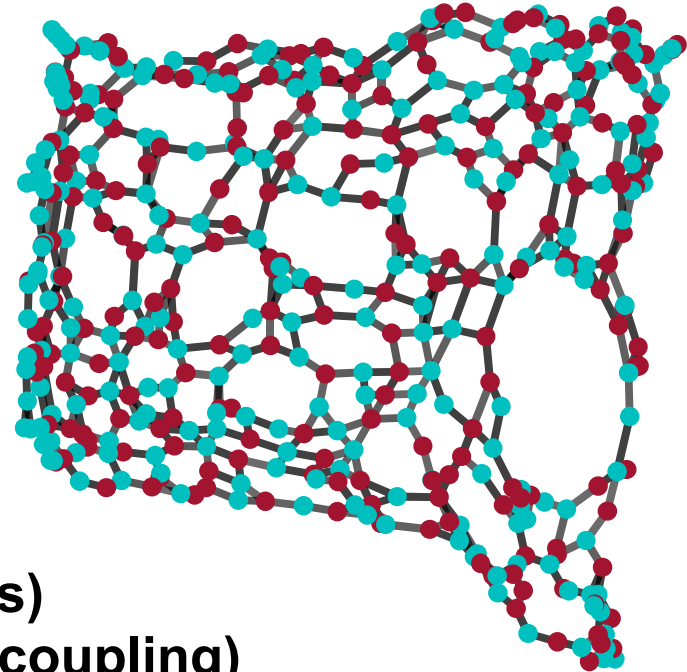
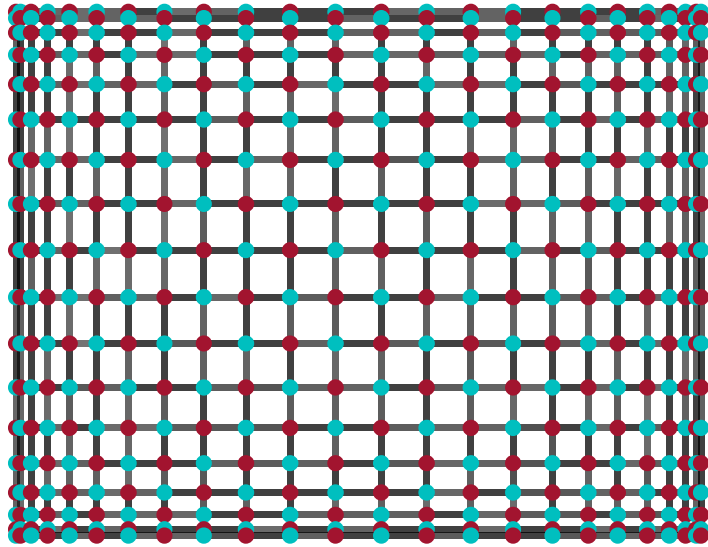
Chip summary

- 28x20=560 coupled oscillators (only limiting factor is chip area)
- Oscillator area < 5% of the full chip area

# Embedding Ising Problem to Hardware

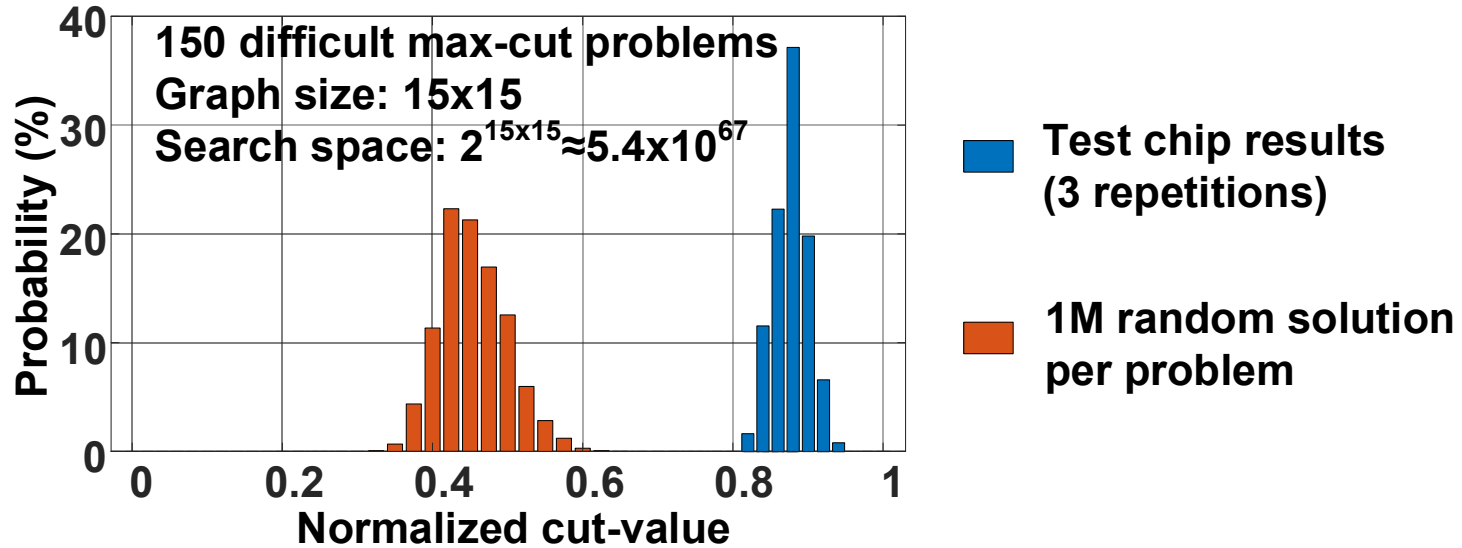


# Example Problems (Regular versus Random Graphs)



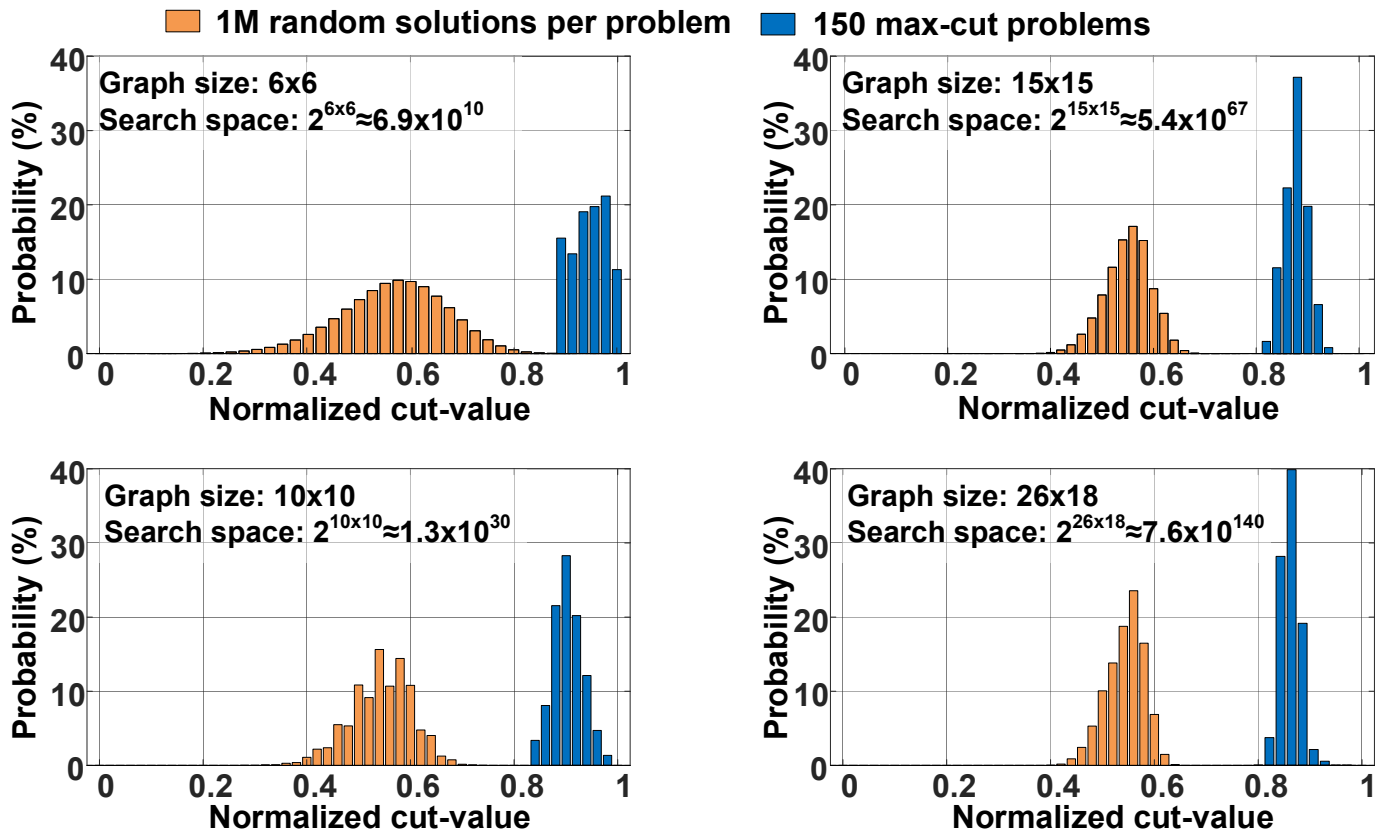
- ● Oscillator (+1 or -1 states)
- B2B inverters (negative coupling)

# Max-cut Results for 15x15 Graphs



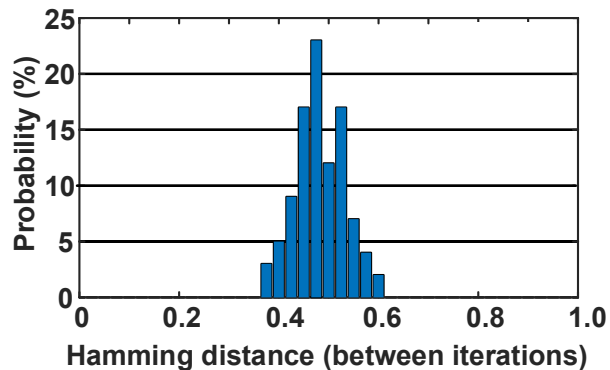
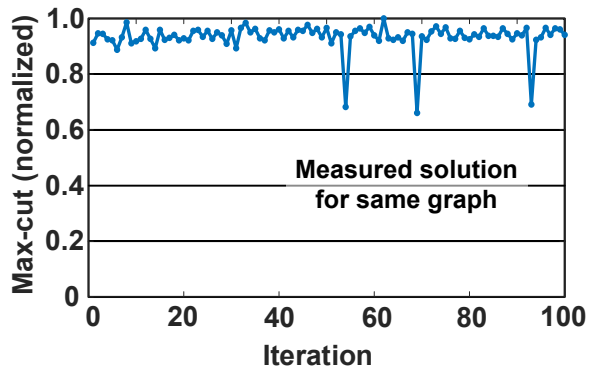
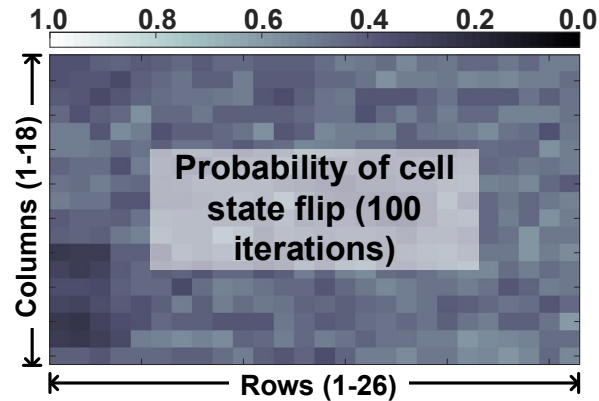
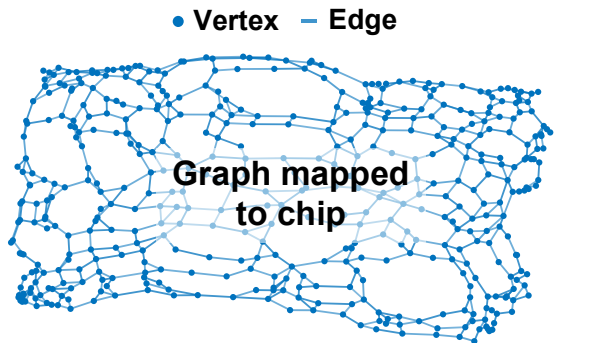
- 150 difficult COPs are mapped and max-cut results are measured for each graph sizes
- Measured results are compared with 1 million randomly sampled solutions from the solution-space for each specific graph.

# Max-Cut Results for Different Graph Sizes

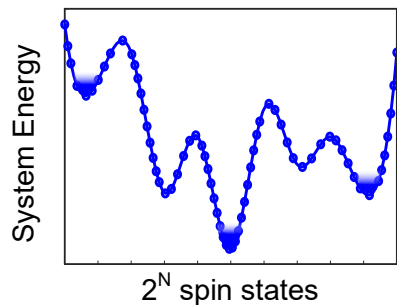
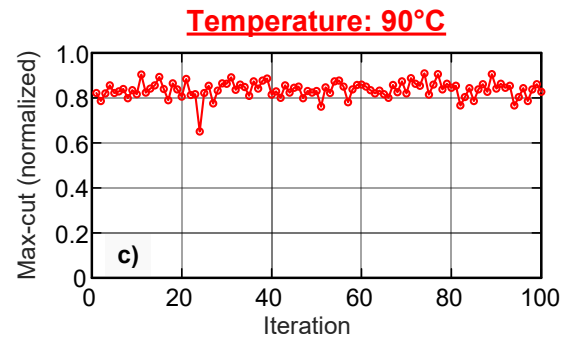
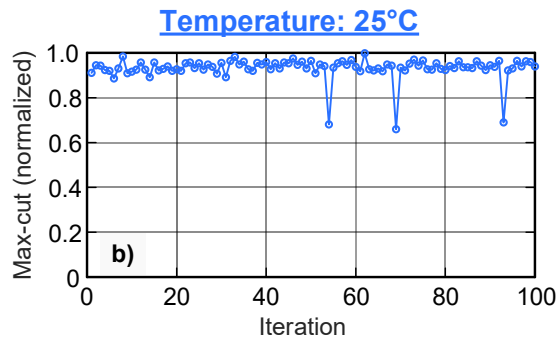
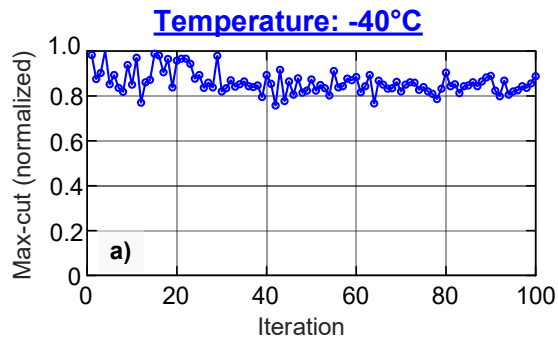




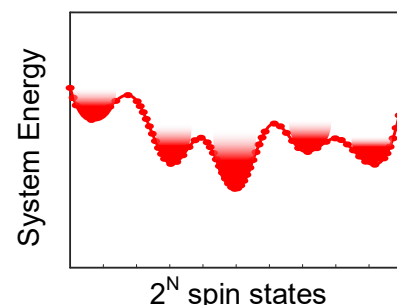
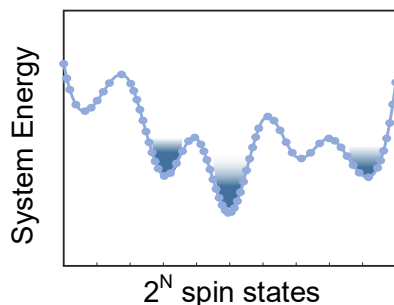
# Repeated Experiment for Same Graph



# Temperature versus Solution Quality

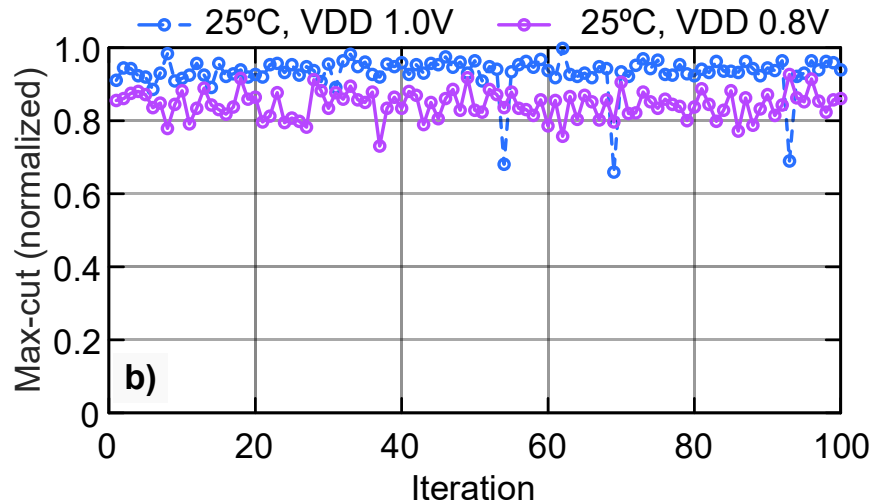
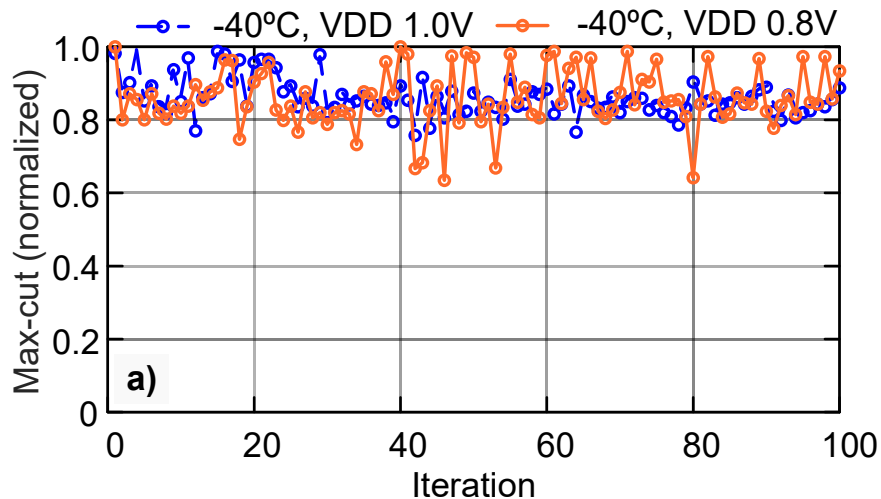


Low temperature:  
lower noise, stronger coupling



High temperature:  
higher noise, weaker coupling

# Supply Voltage versus Solution Quality

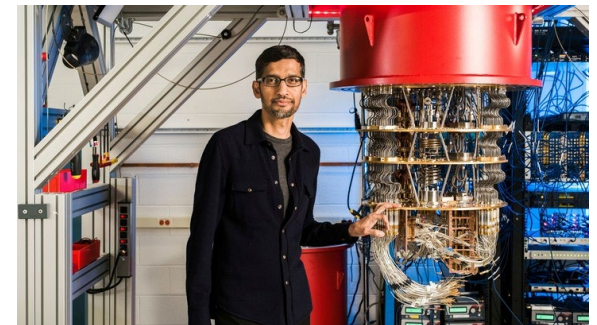
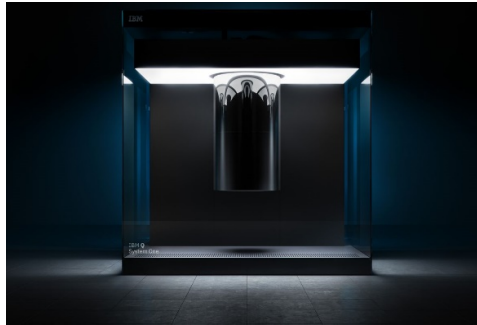


- Lower VDD → higher noise & weaker coupling
- Higher VDD → lower noise & stronger coupling

# Outline

- Introduction to Ising Computers
- Current State of the Art
- Case Study: 560 Coupled Oscillator Test Chip
- **Commercialization Effort and Outlook**
- Summary

# Other Developments: Quantum Computing Startups



## Investments:

Dwave >\$204M

IonQ > \$82M

Rigetti > \$198M

PsiQuantum > \$215M

IBM, Toshiba, Hitachi, Intel, Google, Microsoft,  
NASA, LANL, Amazon AWS, Lockheed, ...

## Challenges:

Kilowatts of cooling power per chip

Large form factor

Limited scalability

Expensive (e.g. \$15M/chip)

Immature technology

# Dwave Quantum Annealer

The screenshot displays the Dwave Quantum Annealer IDE interface. On the left, the Explorer pane shows a file tree for a project named 'CLUSTERING'. The main editor displays a Python script named 'clustering.py' with the following code:


```
64 for coord in coordinates:
65     csp.add_constraint(choose_one_group, (coord.r, coord.g, coord.b))
66
67 # Build initial BQM
68 bqm = dwavebinarycsp.stitch(csp)
69
70 # Edit BQM to bias for close together points to share
71 for i, coord0 in enumerate(coordinates[:-1]):
72     for coord1 in coordinates[i+1:]:
73         # Set up weight
74         d = get_distance(coord0, coord1) / max_distance
75         weight = -math.cos(d*math.pi)
76
77         # Apply weights to BQM
78         bqm.add_interaction(coord0.r, coord1.r, weight)
79         bqm.add_interaction(coord0.g, coord1.g, weight)
80         bqm.add_interaction(coord0.b, coord1.b, weight)
81
82 # Edit BQM to bias for far away points to have difference
83 for i, coord0 in enumerate(coordinates[:-1]):
84     for coord1 in coordinates[i+1:]:
```

The right-hand side of the IDE is divided into two main sections. The top section, 'Problem Details', shows a graph of the problem with nodes and edges. A legend for 'QUBO' indicates that a yellow circle represents a bias of +1 and a white circle represents a bias of 0. The bottom section, 'Solver Details', shows a histogram titled 'Source - Samples - Histogram' with 'SOLUTION OCCURRENCES' on the y-axis and 'ENERGY (SOURCE)' on the x-axis. The histogram shows a distribution of solutions across energy levels, with the highest frequency occurring at the lowest energy level. Below the histogram, the 'Console' pane shows the output of the solver, including the command used to run the script and the resulting plots.

# AWS Braket


## Quantum Processing Units (QPUs)

### D-Wave — Advantage\_system1.1

Quantum Annealer based on superconducting qubits 

Qubits	Status
5760	🟢 ONLINE
Region	Next available
us-west-2	🟢 AVAILABLE NOW

### D-Wave — DW\_2000Q\_6

Quantum Annealer based on superconducting qubits 


Qubits	Status
2048	🟢 ONLINE
Region	Next available
us-west-2	🟢 AVAILABLE NOW

### IonQ

Universal gate-model QPU based on trapped ions 

Qubits	Status
11	🟢 ONLINE
Region	Next available
us-east-1	08:14:38

### Rigetti — Aspen-8

Universal gate-model QPU based on superconducting qubits 

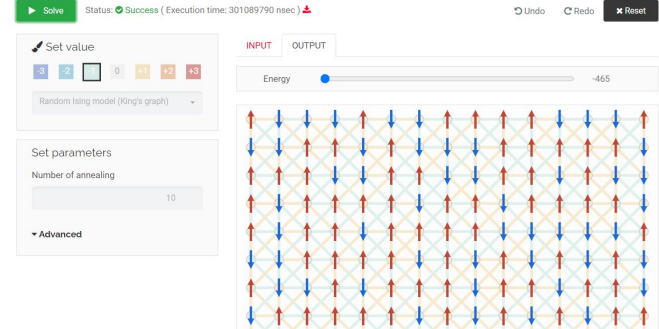
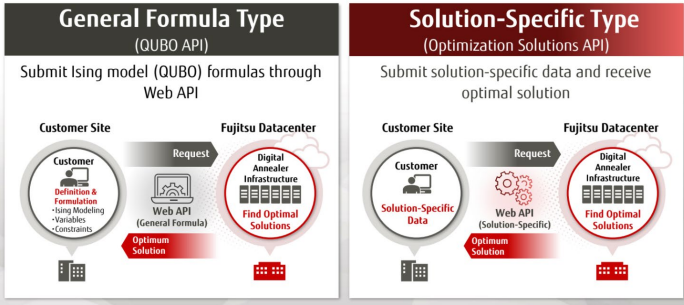
Qubits	Status
31	🟢 ONLINE
Region	Next available
us-west-1	10:14:38

# Fujitsu and Hitachi's Digital Annealers

FUJITSU Quantum-Inspired Computing Digital Annealer  
**Digital Annealer Cloud Service**



[www.annealing-cloud.com](http://www.annealing-cloud.com)



### Quantum Computers

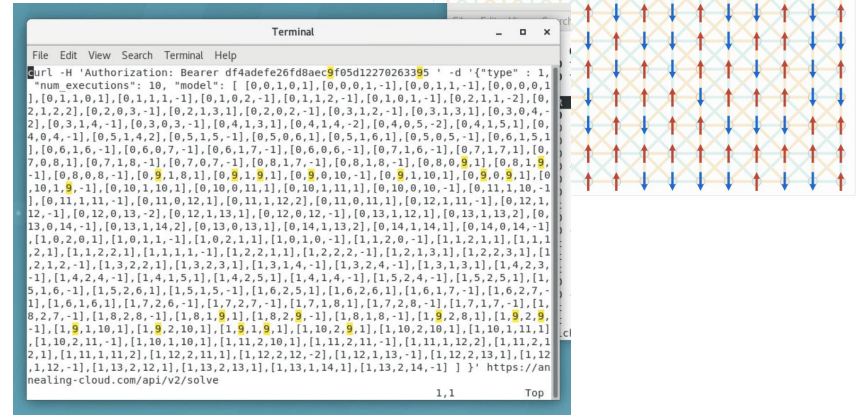
Still in the research stage ...

- Difficult to maintain a quantum state
- Limits in connection and expansion

### Digital Annealer

Easy to apply to actual problems

- Stable operation with digital circuit, and easy miniaturization
- Easy mapping of more complex problems with a fully-connected architecture





# Key Takeaways

- NP-hard optimization problems could be the key driver for future computing growth
- A true coupling based integrated CMOS Ising computer was demonstrated in 65nm
  - Probabilistic exploration of various local minima
  - Mapped and solved 1,000 COPs in the chip with an accuracy of 82%-100%
- For oscillator based computing to be a viable approach however, there has to be a clear and significant power-performance-area advantage over
  - Mathematical optimizers (available today)
  - GPU, FPGA, Custom ASICs, digital annealers (available today)
  - Quantum computers

OBRIGADO  
*gracias*  
 どうも  
 DANKU  
 takk  
 MERCI  
 merci  
 danke schön  
 KÖSZI  
 سپاس  
 PALDIES  
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 ありがとう  
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 MOLTE GRAZIE  
 GO RAIBH MAITH AGAT  
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**invala**  
 謝謝  
**danke**  
 grazas  
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 muchas gracias  
**vielen dank**  
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**شكراً**