Leakage Modeling for Transistors with Steep Sub-threshold Slope Considering Random Threshold Variations

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Outline of Presentation

• Introduction to steep transistors and tunneling FETs

• Statistical leakage estimation of steep transistors

• Monte-Carlo simulation results

• Circuit example: SRAM delay modeling

• Conclusion
MOSFET Leakage Power

- **Challenge:** Sub-threshold current increases exponentially with every generation!

- **Solution:** Reduce threshold voltage by using steep transistor (e.g. Tunneling FET, Ferro-electric FET)

![Graph showing sub-threshold leakage as a function of technology generation.](image)

S. Borkar, Intel Corporation
Tunneling Field Effect Transistor (TFET)

- **ON state:** Positive gate voltage lowers the conduction band edge in the channel.
- **OFF state:** Channel conduction band edge is pushed above the source valence band edge.

TFET $I_d$-$V_g$ Characteristics

$\begin{align*}
I_d &= W \cdot A \cdot E_s \cdot e^{-B/E_s} \\
E_s &= (V_g - V_T)/\lambda \quad \text{where} \ \lambda \ \text{is the effective tunneling distance} \\
\text{Issues: Slope worsens at higher gate voltages, low ON current at high gate voltages}
\end{align*}$
Energy-Performance Comparison: TFET vs. MOSFET

- Sub-threshold slope of TFET is function of $V_{gs}$
- Average sub-threshold slope of TFET is larger than MOSFET
- TFET is energy efficient for frequency below 1GHz

H. Kam, et. al., IEDM 2008
$V_T$ Induced OFF and ON Current Distribution of Tunnel FET

- Monte Carlo simulation assumption: $\sigma_{VT}/\mu_{VT} = 0.1$
- ON current is independent of sub-threshold slope
- OFF current depends on sub-threshold slope
## Statistical Leakage Estimation Techniques

<table>
<thead>
<tr>
<th>Golden</th>
<th>Square-root</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Golden Diagram]</td>
<td>![Square-root Diagram]</td>
<td>![Proposed Diagram]</td>
</tr>
<tr>
<td>$W_y = nW_x$</td>
<td>$W_y = nW_x$</td>
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</tr>
<tr>
<td>$V_{Txi}$: $V_T$ of i-th sub-device</td>
<td>$V_{Ty}$: effective $V_T$</td>
<td>$V_{Ty}$: effective $V_T$</td>
</tr>
</tbody>
</table>

### Given inputs for reference device:

- $W_x$, $\mu_{V_{Txi}}$, $\sigma_{V_{Txi}}$
- $W_y$, $\mu_{V_{Ty}}$, $\sigma_{V_{Ty}}$

### Statistical Leakage Estimation Formulas:

- **Golden Model**
  
  \[
  I_{\text{leak}} \propto \sum_{i=1}^{n} W_x e^{-K_2 V_{Txi}}
  \]

  \[
  \begin{cases}
  \mu(V_{Txi}) = \mu_{V_{Tx}} \\
  \sigma(V_{Txi}) = \sigma_{V_{Tx}}
  \end{cases}
  \]

- **Square-root Model**

  \[
  I_{\text{leak}} \propto W_y e^{-K_2 V_{Ty}}
  \]

  \[
  \begin{cases}
  \mu(V_{Ty}) = \mu_{V_{Tx}} \\
  \sigma(V_{Ty}) = \sigma_{V_{Tx}} / \sqrt{\frac{W_y \cdot L_y}{W_x \cdot L_x}}
  \end{cases}
  \]

- **Proposed Model**

  \[
  I_{\text{leak}} \propto W_y e^{-K_2 V_{Ty}}
  \]

  \[
  \begin{cases}
  \mu(V_{Ty}) = f_{\mu}(W_y, \mu_{V_{Tx}}, \sigma_{V_{Tx}}) \\
  \sigma(V_{Ty}) = f_{\sigma}(W_y, \mu_{V_{Tx}}, \sigma_{V_{Tx}})
  \end{cases}
  \]

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J. Gu, et. al., DAC 2007
Leakage Estimation Error Using Square Root Method

- Mean value of $V_T$ of device to be modeled is same as the mean value of the reference device
- % error increases with steeper sub-threshold slope
Leakage Estimation: Wilkinson’s Method

• Basic Premise: Sum of log-normal distributions of a number of random variables can be expressed as a single log-normal distribution

• Define variables:

1. Mean and standard deviation of reference Gaussian random variable \( X_i \) are \((m_{X_i}, \sigma_{X_i})\)

2. Mean and standard deviation of Gaussian random variable to be estimated \( Y \) are \((m_Y, \sigma_Y)\)

3. \( r_{ij} \) is the correlation coefficient between random variables \( X_i \)

4. Define log-normal distributions corresponding to \( Y \) and \( X_i \)

\[ e^y \quad \text{and} \quad \sum_{i=1}^{n} \frac{1}{n} e^{X_i} \]
Wilkinson’s Method Contd.

• Equate first and second moment of two log-normal distributions:

1. \[ \sum_{i=1}^{n} \frac{1}{n} e^{m_{x_i} + \frac{\sigma_{x_i}^2}{2}} = e^{m_y + \frac{\sigma_y^2}{2}} \] (1\textsuperscript{st} moment)

2. \[ \frac{1}{n^2} \left( \sum_{i=1}^{n} e^{2m_{x_i} + 2\sigma_{x_i}^2} \right) = e^{2m_y + 2\sigma_y^2} \] (2\textsuperscript{nd} moment)

• Solve for \( m_y \) and \( \sigma_y \):

1. \( m_y = m_x + \frac{1}{2} \Delta \) and 2. \( \sigma_y^2 = \sigma_x^2 - \Delta \)

where \( \Delta = \sigma_x^2 - \ln \left( \frac{e^{\sigma_x^2} + (n - 1) \cdot e^{r \cdot \sigma_x^2}}{n} \right) \)
Proposed Leakage Estimation Technique

- Based on Wilkinson’s approach of moment matching of log-normally distributed random variables
- Takes spatial correlation between reference devices into account
- Matches golden leakage distribution almost accurately

\[
\begin{align*}
\mu_{V_{Ty}} &= \mu_{V_{Tx}} - \frac{1}{2} \frac{\Delta}{K^2} \\
\sigma^2_{V_{Ty}} &= \sigma^2_{V_{Tx}} - \frac{\Delta}{K^2} \quad (\Delta \geq 0)
\end{align*}
\]

Where \( \Delta = K^2 \sigma^2_{V_{Tx}} - \ln\left(\frac{e^{K^2 \sigma^2_{V_{Tx}}} + (n-1) \cdot e^{rK^2 \sigma^2_{V_{Tx}}}}{n}\right) \geq 0 \)
Monte-Carlo Simulation Results (Different Device Widths)

W = 5W_x, $\sigma_{VTx}/\mu_{VTx}$ = 10%, r=0

<table>
<thead>
<tr>
<th></th>
<th>Square Root</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>3σ error in leakage</td>
<td>22.21%</td>
<td>0.99%</td>
</tr>
<tr>
<td>6σ error in leakage</td>
<td>31.52%</td>
<td>10.91%</td>
</tr>
</tbody>
</table>

W = 10W_x, $\sigma_{VTx}/\mu_{VTx}$ = 10%, r=0

<table>
<thead>
<tr>
<th></th>
<th>Square Root</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>3σ error in leakage</td>
<td>23.58%</td>
<td>1.11%</td>
</tr>
<tr>
<td>6σ error in leakage</td>
<td>30.47%</td>
<td>5.98%</td>
</tr>
</tbody>
</table>
Monte-Carlo Simulation Results
(Different $V_T$ Variations)

<table>
<thead>
<tr>
<th>$W=5W_x$, $\sigma_{VTx}/\mu_{VTx}=5%$, $r=0$</th>
<th>$W=5W_x$, $\sigma_{VTx}/\mu_{VTx}=10%$, $r=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Square Root</strong></td>
<td><strong>Proposed</strong></td>
</tr>
<tr>
<td><strong>3(\sigma) error in leakage</strong></td>
<td>5.36%</td>
</tr>
<tr>
<td><strong>6(\sigma) error in leakage</strong></td>
<td>6.50%</td>
</tr>
</tbody>
</table>
Monte-Carlo Simulation Results (Different Correlation Coefficients)

<table>
<thead>
<tr>
<th>W=10Wₓ, σₓ/μₓ=10%, r=0.1</th>
<th>W=10Wₓ, σₓ/μₓ=10%, r=0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td><strong>Square Root</strong></td>
<td><strong>Proposed</strong></td>
</tr>
<tr>
<td><strong>3σ error in leakage</strong></td>
<td>33.61%</td>
</tr>
<tr>
<td></td>
<td>0.49%</td>
</tr>
<tr>
<td><strong>6σ error in leakage</strong></td>
<td>42.58%</td>
</tr>
<tr>
<td></td>
<td>2.37%</td>
</tr>
<tr>
<td><strong>Square Root</strong></td>
<td><strong>Proposed</strong></td>
</tr>
<tr>
<td><strong>3σ error in leakage</strong></td>
<td>50.58%</td>
</tr>
<tr>
<td></td>
<td>0.59%</td>
</tr>
<tr>
<td><strong>6σ error in leakage</strong></td>
<td>64.60%</td>
</tr>
<tr>
<td></td>
<td>0.30%</td>
</tr>
</tbody>
</table>
Circuit Example: SRAM Bitline Delay

- SRAM bitline read delay increases due to leakage current of unaccessed cells
- Worst case sensing delay: accessed cell stores a ‘1’ and all other cells store ‘0’
For bitline delay modeling Monte Carlo simulations were performed assuming $\sigma_{VT}/\mu_{VT} = 0.1$

Proposed method shows less than 1% error in 3$\sigma$ bitline delay estimation
Conclusions

• Devices with steep sub-threshold slope are vulnerable to leakage variation due to random $V_T$ shift

• Wilkinson’s approach estimate leakage variation in steep SS devices with high accuracy

• Monte Carlo simulations shows less than 11% error in estimating $6\sigma$ leakage current

• Worst-case error in $3\sigma$ SRAM bitline delay estimation using our model is less than 0.5%.