BASIC PROBABILITY THEORY

EVENTS: All events will be modeled as sets where a set is a collection of outcomes of experiments or trials. The set of outcomes can be generated by repeating the same experiment on a component or system over and over again or by running the same experiment on a large number of components.

The set of all possible outcomes is the SAMPLE SPACE and is designated using the greek $\Omega$, then an event is a subset of $\Omega$ and consists of all outcomes that satisfy some property.

PROBABILITY: is a measure assigned to an event. It is a real valued function on the interval 0 to 1 including 0 and 1.

AXIOMS OF PROBABILITY:
I) Prob of and event, E is designated $P(E)$, where $0 \leq P(E) \leq 1$ and $P(\Omega) = 1$

II) if an event $E_a$ and an event $E_b$ are disjoint, meaning they share no outcomes, then

$$ P(E_a \cup E_b) = P(E_a) + P(E_b) $$

where the U operator is called Union operator and is also spoken of with as the OR operator since we say we are obtaining the probability of an event that is satisfied by $E_a$ OR $E_b$.

The most common means of obtaining $P(E)$ is to observe a sufficient number of outcomes of a test or experiment and record the number satisfying E. then:

$$ P(E) = \lim_{n \to \infty} \left( \frac{n_E}{n} \right) $$

This is called the “relative frequency” approach.

CONDITIONAL PROBABILITY
This refers to events which are related in some way so that we can speak of the “probability of event A given that event B is known to have occurred”. Thus we could ask about the probability that a component would fail in the next year, given that it had been maintained properly.

This is written as $P(A|B)$ or “Probability of A given B”
The formula for $P(A \mid B)$ is:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

where the $\cap$ operator is the INTERSECTION OPERATOR and requires BOTH A and B to be true, and therefore is also referred to as the AND operator. We can rework the expression above as:

$$P(A \cap B) = P(A \mid B)P(B)$$

which says that the probability of having both A and B events is the probability of having A given B times the probability that B is true.

INDEPENDENT EVENTS
Often in reliability analysis we ask if events are independent. The failure of a transformer and the event that it was struck by lightning are most likely not independent events. But the event that a transformer failed at the same time as another transformer from a different manufacturer failed, are most likely independent events (unless we can establish that some third event was true for both transformers).

The probability of two independent events both happening is $P(A \cap B)$ and this is equal to $P(A)P(B)$

The probability of either A or B or both events for two independent events $P(A \cup B)$ is equal to $P(A) + P(B) - P(A)P(B)$ Where the third term is necessary to cover the “both”.

MUTUALLY EXCLUSIVE EVENTS

We say events are mutually exclusive if they are made up of completely different events that cannot occur at the same time. If events A and B are mutually exclusive then

$$P(A \cap B) = P(A \mid B)P(B) = 0$$

This is true because $P(A \mid B)$ must itself be zero by our definition of mutually exclusive (if B is true then A cannot be true).
Finally $P(A \cup B) = P(A) + P(B)$ for the mutually exclusive case.

RANDOM VARIABLES:

Real Valued random Variables
If $x$ is a real number, we can ask questions about the probability of events such as: $X \leq x$ or
$P(X \leq x)$ In fact this probability is called the Cumulative Distribution Function or CDF of $X$

$$F_X(x) = P(X \leq x)$$

clearly $0 \leq F_X(x) \leq 1$ and the function $F_X(x)$ is a non decreasing function.

Note that we cannot speak of the probability of a real numbered random variable exactly equaling a real value. The probability that a real numbered random variable equalling exactly 0.15500000 is zero. But we can express the probability that the random variable is less than 0.15500000 or that it is between 0.15499995 and 0.15500000 etc.

This is formalized into the relationship:

$$f_X(x) = \lim_{\Delta x \to 0} \frac{P[x < X \leq x + \Delta x]}{\Delta x}$$

where the function $f_X(x)$ is called the Probability Density Function or PDF (also sometimes simply called the “density”). These relationships hold:

$$\int_{-\infty}^{\infty} f_X(x) dx = 1$$

$$F_X(x) = \int_{-\infty}^{x} f_X(x) dx$$
Discrete random variables:
When the random variable is discrete - that is - it only takes on integer values then we can speak of the probability of it exactly equal to one of the possible integer values. We use lower case $p$ for this so that

$$p_X(x) = P(X=x)$$

and

$$F_X(x) = \sum_{x_i \leq x} p_X(x_i)$$

We shall now begin to apply these definitions to reliability analysis.